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ANALYSIS AND DESIGN OF A CONTINUOUS PRESTRESSED CONCRETE GIRDER

[illegible] JOHN GRATAN DEVLIN AND
[illegible] ALLEN FREDERICK DILL

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A THESIS SUBMITTED TO THE FACULTY
OF RENSSELAER POLYTECHNIC INSTITUTE
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR DEGREE OF
MASTER OF CIVIL ENGINEERING

by

[illegible]

and

[illegible]

Troy, New York

May, [illegible]

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OF A
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BY
JOHN GRATAN DEVLIN
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ALLEN FREDERICK DILL
TROY, NEW YORK
MAY, 1950

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12844

TABLE OF CONTENTS

SECTION I - INTRODUCTION Page

A. Object of Thesis 1
B. Introduction and History 2

SECTION II - THEORY OF PRESTRESSED CONCRETE GIRDERS

A. Problem of Continuous Beam 6
B. Symbols 8
C. Development of Fundamental Formulae 10
D. Secondary Bending Moment at an Interior Support 19

SECTION III- GENERAL FEATURES

A. Foundations and Abutments 22
B. Construction Procedure 23
C. General Features of Design 24
D. Specifications 27
E. Properties of Materials 28

SECTION IV - DESIGN LOADS

A. Loading for Moment 30
  1. Moment Distribution 30
  2. Reactions 34
  3. Influence Line Ordinates for Moment 35
  4. Design Moments 38
B. Loading for Shear 40
  1. Influence Line Ordinates for Shear 41
  2. Design Shear 42

SECTION V - GIRDER DESIGN

A. Cross Section 44
B. Determination of Wire Area 53
C. Design of Bearing Plates - Longitudinal Wires 54
D. Shear Check 55
E. Web Reinforcement 57
F. Anchor Blocks 58
G. Anchor Block Reinforcement 59
H. Shear Key 61
I. Transverse Wires 62
J. Design of Bearing Plates - Transverse Wires 63
K. Moment at First Crack 64
L. Deflection 65
M. Comparison of Section Designed - Straight Cables versus Parabolic Cable 67

SECTION VI - CONCLUSIONS AND RECOMMENDATIONS 69

SECTION VII- BIBLIOGRAPHY 70

OBJECT OF THESIS

Though prestressed concrete girder bridges are relatively new in this country, design procedure for simply supported beams have been published by Gustav Magnel, P.W. Abeles and others. On the subject of continuous girders, however, there has not been too much published. Magnel has published an analysis and set forth a procedure for the design of continuous girders in his "Prestressed Concrete". In his design, Magnel uses a continuous parabolic cable. This cable introduces secondary bending moments which results in an involved design procedure employing a complicated graph.

The object of this thesis is to analize and develop a design procedure for a three equal span continuous girder bridge which employs a simple cable arrangement.

INTRODUCTION AND HISTORY

Prestressing concrete is not a new development in the field of Reinforced Concrete. As early as 1888, Doehonig took out a patent in Berlin for mortar slabs reinforced by prestressed steel wires exercising a permanent compression on the tension zone of the concrete. Poor quality cement was blamed for the failure. Lund and Koenen early in the twentieth century tested prestressed reinforced concrete beams by prestressing to approximately 8,000 lbs. per square inch. Because of the low pretension of the reinforcement, after a time the prestressing disappeared altogether. Other early attempts at prestressing failed because the steel reinforcement used had too low a yield point and the pretension was too small to overcome the shrinkage and creep of the concrete.

The present day investigator has at hand reinforcement with ultimate strengths well over 200,000 lbs. per square inch and concrete with crushing strengths approaching 10,000 lbs. per square inch. Hence, he is in a position to design prestressed reinforced concrete structures with steel and concrete of a quality to give designs taking advantage of the principles of prestressing and the higher strengths of steel and concrete.

In the field of bridge design especially, prestressed designs result in great savings of materials, permit considerably longer spans, more reliable structures as the tensile stresses are completely cancelled and cracks are

prevented from occuring as the entire cross-section is stressed compressively. Consequently, a number of methods of prestressing girders have appeared within the past two decades.

The first practical solution was by Freyssinet in 1928 utilizing high-quality concrete and high-tensile steel. To derive the dense and exceptional quality of concrete required, it was essential to use every available means of compaction such as vibration, heating, and so forth. Concrete under Freyssinet's treatment resulted after a few hours in a concrete made with ordinary portland cement withstanding a crushing stress of 4,000 to 6,000 lbs. per square inch and after hardening, a final compressive strength of 150,000 lbs. per square inch. Freyssinet employed wires of 0.2" diameter placed against the internal face of a sheath and positioned by short lengths of helical springs (especially where the cable changed directions). A hydraulic jack is used to stress the wires and they are finally secured by cylindrical blocks. The advantages claimed for this method are an inexpensive method of securing the wires, the wires are quickly stretched, the end securings for the wires do not protrude beyond the end of the beam. Disadvantages which may be claimed are that the wires may not be equally stressed, that the shape and quality of the end blocks possibly may not be uniform, and the necessity of heavy and expensive jacks. This process was responsible for a great deal of work in France prior to 1939 but has proved too cumbersome for general acceptance. Since then, several pre-

stressing methods have been developed that are much simpler and considerably more economical.

During this past war it was practically impossible to obtain the Freyssinet devices in Belgium and another method variously called the Belgium Method, the Magnel Method, or the Magnel-Blaton Method was developed by Gustav Magnel. In this method wires are placed in a definite order with a spacing equal to the wire diameter for grouting purposes. The wires are stretched two at a time and are strongly anchored to devices called "sandwich plates". The wires are enclosed within a sheet-metal sheath and tensioned by a five-ton jack. The advantages claimed for this method are a cable of a large number of wires, wires are individually stretched and also tested to a stress approximately 10% higher than under working loads. Disadvantages stated are that it is more expensive than Freyssinet, a longer time to tension the prestressing wires, extension of the "sandwich plates" beyond the end of the concrete, and more difficult to bend the cable to anchor it. A considerable amount of work has been completed in Belgium and appears to be very satisfactory.

Two American processes have been developed. The Electrical Method (Billner and Carlson) employs threaded steel bars coated with sulphur which upon being heated electrically elongate. Nuts are then tightened down and the bars allowed to cool, with the bond re-established when the sulphur solidifies, allowing the nuts now to be removed. The whole operation of extending one rod can be completed in about two minutes. Disadvantages set forth are the wastage of steel

area unless the ends of the bars are upset, serious loss of prestress due to small stress, possible lack of uniformity in the prestress and possible chemical action of the sulphur damaging the steel and concrete (especially if cracks appear and moisture is present). However, this method is simple, economical and practical, and merits investigation and use.

The second American method, Schorer's, depends upon the bond of thin wires, wound helically in opposite directions around a central core forming a cage. The wires are separated from one another and the central core by spacer discs, being held in place at the desired stress by a wedge-ring clamping device. Units can be manufactured in various sizes depending on the wire size and central core. The advantages claimed for this system is the use of a factory-made unit requiring no special treatment in the field.

Processes depending upon the expansion of concrete in hardening have been investigated but full reports are not yet available.

PROBLEM OF THE CONTINUOUS BEAM

Prior to discussing the problem of continuous beams, it may be well to adopt a convention of signs. A positive bending moment causes compressive stress in the top fibre (i.e., causes the beam to sag) and negative bending moment causes tensile stress in the top fibre (i.e., causes the beam to hump). The eccentricity is designated as follows: where the cable is above the neutral axis as positive and where below the neutral axis as negative. The convention thus established for the bending moment and eccentricity agree with each other.

In a continuous beam unless the prestressing cable coincides with the neutral axis of the beam (i.e., is nowhere eccentric), the statically indeterminate reactions are altered by the prestressing.

If in Fig. 1, the support at B is removed, the prestressing force would create a negative bending moment and tend to lift the beam at all points except at the exterior supports, A and C. Therefore, to keep the beam in contact with the support at B (assume support replaced), a force F_B must be applied. The application of this force produced an extra upward reaction at the exterior supports (the sum of which must equal the force applied). These additional reactions produce an additional bending moment - which is designated as the Secondary Bending Moment, M_s, which must be combined with the initial bending moments caused

PROBLEM OF THE CONTINUOUS BEAM

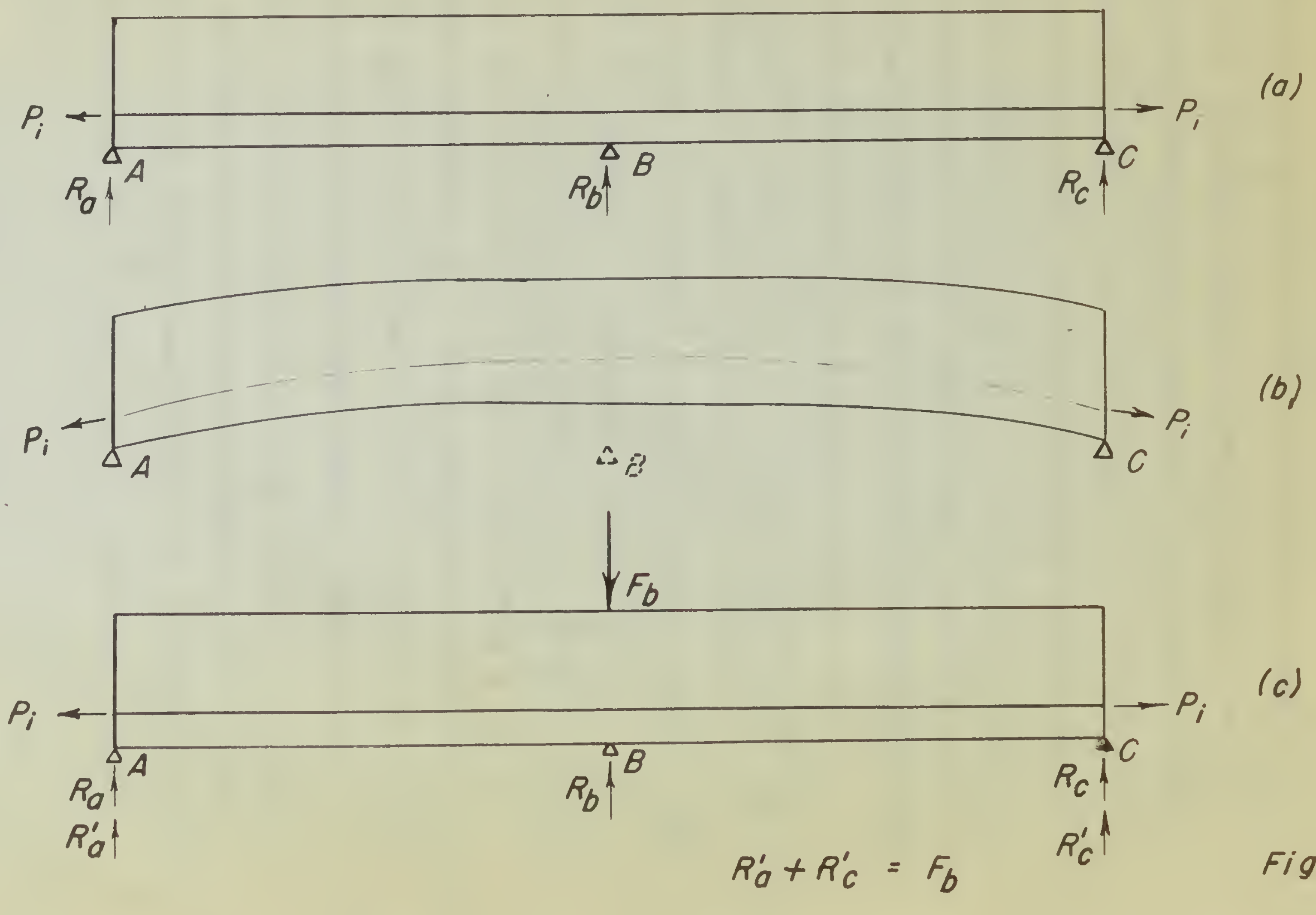


Fig. 1

by the loads and the initial prestressing force, P_i.

This Secondary Bending Moment is proportional to the initial prestressing force and is not negligible (unless the cable is so placed that the eccentricities are such that they partially cancel one another throughout the length of the beam). The magnitude and sign of the Secondary Bending Moment result from the position of the cable, which in turn can only be chosen when the values of the Secondary Bending Moment are known.

To avoid the process of repeated calculations for the Secondary Bending Moment and the position of the cable, which are mutually dependent, Magnel suggests the use of the concept of equivalent eccentricities. He defines the equivalent eccentricity at a section as the sum of the actual eccentricity at that section and an apparent eccentricity. This apparent eccentricity is introduced by the Secondary Bending Moment and is equal to the value of the Secondary Bending Moment at the section divided by the prestressing force; i.e., $M_s \div P_i$.

Therefore, if the equivalent eccentricity is used instead of the actual eccentricity, the continuous beam can be designed as if there were no Secondary Bending Moment.

SYMBOLS

A - cross-sectional area of the beam

A, B, C, D - designate sections (when used as subscripts)

A_v - area of steel reinforcement

b - width of section at centroid

c, c_t - allowable compressive and tensile stress in the concrete, respectively

e - eccentricity of the prestressing cable from the neutral axis

e' - equivalent eccentricity

f_{dt}, f_{db} - stress in top and bottom fibre, resp., caused by w_d

f_{at}, f_{ab} - stress in top and bottom fibre, resp., caused by w_a

f'_{at}, f'_{ab} -stress in top and bottom fibre, resp., caused by w'_a

f_v - allowable tensile stress of steel reinforcement

f_{cr} - allowable concrete tensile stress (for check on cracking)

I - moment of inertia of entire section about centroid

L - length of each span, feet

M_d - bending moment due to w_d

M_a - bending moment due to w_a

M'_a - bending moment due to w'_a

n - proportion of P_i that remains permanently; generally = 0.85

P_i - initial prestressing force

Q - statical moment of section on either side of centroid taken about the centroid

r - radius of gyration of concrete section

St - principle stress

V - external shear on section

v - shearing stress at the centroid

w_d - load per unit length acting when prestress is being established

w_a - additional load per unit length acting after the prestress has been established (acting in such a manner as to produce a moment of same sign as that produced by w_d)

w'_a - additional load per unit length acting after the prestress has been established (acting in such a manner as to produce a moment of opposite sign as that produced by w_d)

y_t, y_b - distances from centroid of beam to top and bottom fibres, respectively

DEVELOPMENT OF FUNDAMENTAL FORMULAE

A prestressed concrete member subjected to bending only must in general resist the bending moment produced by the loads present (w_d) when the prestress is being established and to another bending moment produced by the loads (w_a and w'_a) which can be applied after the prestressing.

The cases of a simply supported beam subjected to bending moments of the same sign (M_d and M_a), a simply supported beam subjected to bending moments of opposite sign (M_d and M'_a), and a continuous beam subjected to bending moments of the same and opposite sign (M_d, M_a and M'_a). Each case will be investigated at the sections of maximum moment, for the top and bottom fibre, and at the two critical loading conditions - immediately after the prestress is established and after an elapsed time.

I. Simply supported beam subjected to bending moments of the same sign.

A. Case of eccentricity greater than r^2/y_t

1. Top fibre at mid-beam section.

a. Immediately the prestress is established the tensile stress in the concrete under the load w_d and the prestressing force must not exceed the permissible tensile strength (c_t) in the concrete.

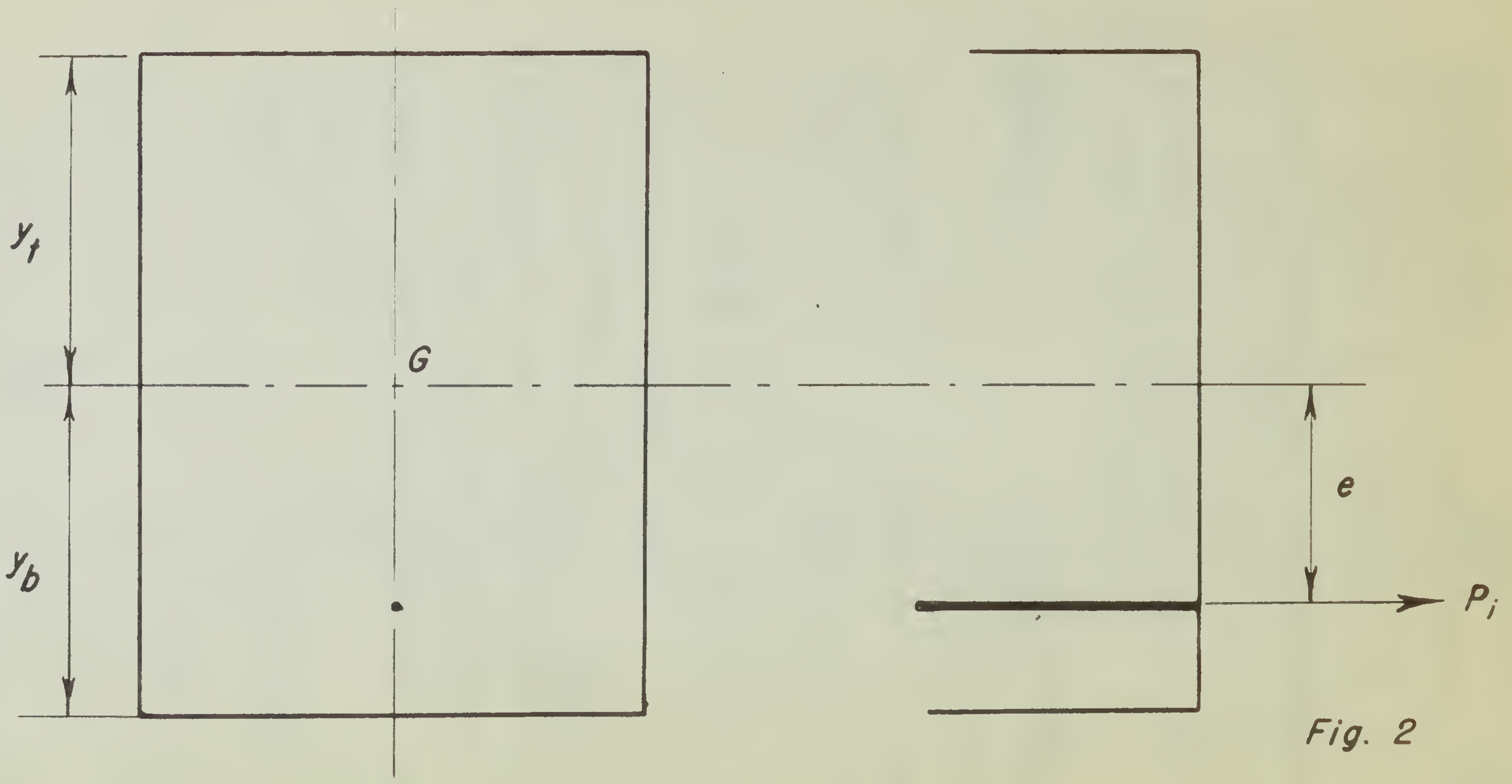


Fig. 2

CONDITIONS FOR THE
DEVELOPMENT OF
FUNDAMENTAL FORMULAE

FORCE	STRESS CAUSED	
Prestress	P_i/A	(compression)
	$P_i(ey_t)/Ar^2$	(tension)
Dead Load (w_d)	f_{dt}	(compression)

$$\frac{P_i}{A}\left(\frac{ey_t}{r^2} - 1\right) - f_{dt} \leqq c_t \qquad (1)$$

b. After an elapsed time the compressive stress in the concrete under the loads (w_d and w_a) and the prestressing force must not exceed the compressive strength (c) of the concrete.

FORCE	STRESS CAUSED	
Prestress	nP_i/A	(compression)
	$nP_i(ey_t)/Ar^2$	(tension)
Dead Load (w_d)	f_{dt}	(compression)
Live Load (w_a)	f_{at}	(compression)

$$\frac{-nP_i}{A}\left(\frac{ey_t}{r^2} - 1\right) + f_{dt} + f_{at} \leqq c \qquad (2)$$

2. Bottom fibre at mid-beam section.

a. Immediately the prestress is established the compressive stress in the concrete under the load (w_d) and the prestressing force must not exceed the permissible compressive strength in the concrete.

FORCES	STRESS CAUSED	
Prestress	P_i/A	(compression)
	$P_i(ey_b)/Ar^2$	(compression)
Dead Load (w_d)	f_{db}	(tension)

$$\frac{P_i}{A}\left(1 + \frac{ey_b}{r^2}\right) - f_{db} \leqq c \qquad (3)$$

b. After an elapsed time the tensile stress in the concrete under the loads (w_d and w_a) and the prestressing force must not exceed the permissible tensile strength in the concrete.

FORCE	STRESS CAUSED	
Prestress	nP_i/A	(compression)
	$nP_i(ey_b)/Ar^2$	(compression)
Dead Load (w_d)	f_{db}	(tension)
Live Load (w_a)	f_{ab}	(tension)

$$\frac{-nP_i}{A}\left(1 + \frac{ey_b}{r^2}\right) + f_{db} + f_{ab} \overset{<}{=} c_t \qquad (4)$$

B. Case of eccentricity less than r^2/y_t

1. Top fibre at mid-beam section.

a. Immediately the prestress is established the condition of I-A-1-a does not apply since the top fibre is now always in compression. (5)

Therefore, the controlling condition after prestress is that the compressive stress under the loads (w_d and w_a) and the prestressing force must not exceed the compressive strength of the concrete.

FORCE	STRESS CAUSED	
Prestress	P_i/A	(compression)
	$P_i(ey_t)/Ar^2$	(tension)
Dead Load (w_d)	f_{dt}	(compression)
Live Load (w_a)	f_{at}	(compression)

$$\frac{P_i}{A}\left(1 - \frac{ey_t}{r^2}\right) + f_{dt} + f_{at} \overset{<}{=} c \qquad (6)$$

b. After an elapsed time the compressive stress under the loads (w_d and w_a) and the prestressing force must not exceed the compressive strength of the concrete. All the stresses remain the same as those of I-B-1-a except the P_i terms which are reduced by n; therefore, if (6) is satisfied, this condition is also. (6-a)

2. Bottom fibre at mid-beam section.

a. Immediately the prestress is established and after
b.
an elapsed time the conditions are the same as in I-A-2-a and -b. Therefore, the condition equations (3) and (4) apply (redesignate as (7) and (8), respectively).

II. Simply supported beam subjected to bending moments of the opposite sign.

A. Case of eccentricity greater than r^2/y_t

1. Top fibre at mid-beam section.

a. Immediately the prestress is established the tensile stress in the concrete under the loads (w_d and w'_a) and the prestressing force must not exceed the permissible tensile strength of the concrete.

FORCE	STRESS CAUSED	
Prestress	P_i/A	(compression)
	$P_i(ey_t)/Ar^2$	(tension)
Dead Load (w_d)	f_{dt}	(compression)
Live Load (w'_a)	f^{8}_{at}	(tension)

$$\frac{P_i}{A}\left(\frac{ey_t}{r^2} - 1\right) - f_{dt} + f'_{at} \leq c_t \qquad (9)$$

b. After an elapsed time the condition and forces are the same as those of II-A-1-a except the P_i terms which are reduced by n; therefore, if (9) is satisfied, this condition is also. (9-a)

2. Bottom fibre at mid-beam section.

a. Immediately the prestress is established the compressive stress in the concrete under the loads (w_d and w'_a) and the prestressing force must not exceed the compressive strength of the concrete.

FORCE	STRESS CAUSED	
Prestress	P_i/A	(compression)
	$P_i(ey_b)/Ar^2$	(compression)
Dead Load (w_d)	f_{db}	(tension)
Live Load (w'_a)	f'_{ab}	(compression)

$$\frac{P_i}{A}\left(1 + \frac{ey_b}{r^2}\right) - f_{db} + f'_{ab} \leq c \qquad (10)$$

b. After an elapsed time the condition and forces are the same as those of II-A-2-a except the P_i terms which are reduced by n; therefore, if (10) is satisfied, this condition is also. (10-a)

B. Case of eccentricity less than r^2/y_t

1. Top fibre at mid-beam section.

a. Immediately the prestress is established the tensile stress in the concrete under the loads (w_d and w'_a) and the prestressing force must not exceed the permissible tensile strength of the concrete.

FORCE	STRESS CAUSED	
Prestress	P_i/A	(compression)
	$P_i(ey_t)/Ar^2$	(tension)
Dead Load (w_d)	f_{dt}	(compression)
Live Load (w'_a)	f'_{at}	(tension)

$$\frac{-P_i}{A}\left(1 - \frac{ey_t}{r^2}\right) - f_{dt} + f'_{at} \leq c_t \qquad (11)$$

b. After an elapsed time the condition and forces are the same as those of II-B-1-a except the P_i terms which are reduced by n; therefore, this condition controls.

$$\frac{-nP_i}{A}\left(1 - \frac{ey_t}{r^2}\right) - f_{dt} + f'_{at} \leqq c_t \qquad (11\text{-a})$$

2. Bottom fibre at mid-beam section.

a. Immediately the prestress is established the compressive stress in the concrete under the loads (w_d and w'_a) and the prestressing force must not exceed the compressive strength of the concrete.

FORCE	STRESS CAUSED	
Prestress	P_i/A	(compression)
	$P_i(ey_b)/Ar^2$	(compression)
Dead Load (w_d)	f_{db}	(tension)
Live Load (w'_a)	f'_{ab}	(compression)

$$\frac{P_i}{A}\left(1 + \frac{ey_b}{r^2}\right) - f_{db} + f'_{ab} \leqq c \qquad (12)$$

b. After an elapsed time the condition and forces are the same as those of II-B-2-a except the P_i terms which are reduced by n; therefore, if (12) is satisfied, this condition is also. (12-a)

NOTE: It is obvious that the beam must satisfy the formulae derived above in I when the beam is loaded thusly. However, if the conditions of certain of these equations are satisfied, those of others are automatically satisfied.

Therefore, for a beam subjected to bending moments of the same and opposite sign, whether the beam be simply supported or continuous, there are the controlling formulae for the section or sections of maximum bending moment or moments, respectively.

For Case A ($e > r^2/y_t$) -

if (9) is satisfied, (1) is also;
if (10) is satisfied, (3) is also.

Therefore, the controlling equations are -

(2), (9), (4) and (10).

For Case B ($e < r^2/y_t$) -

if (12) =(10) is satisfied, (3) is also and (1) is inoperative.

Therefore, the controlling equations are -

(6),(11), (4) and (10).

The next consideration is the graphical representation of the controlling formulae for beams subjected to bending moments of the same and opposite sign.

GRAPHICAL REPRESENTATION OF FORMULAS

To facilitate the solution of the four simultaneously controlling formulae it will be worthwhile to represent the formulas graphically. Each formula will be considered individually and then simultaneously with the three companion condition formulas for the solution of sections of maximum moment.

The formulae are plotted with the eccentricity as the abscissa and the value of $(1/P_i)$ as the ordinate. The value of the abscissa (e) for the ordinate $(1/P_i)$ equal to zero for lines 2 and 2' is $+r^2/y_t$ and for the lines 4 and 4' is $-r^2/y_b$. The value of the ordinate $(1/P_i)$ for the abscissa (e) equal to zero is given in the following table. (see Fig. 3 for graphical representation)

FORMULA	LINE	ABSCISSA	ORDINATE	CONDITION	
(6)	2	$+r^2/y_t$	$\frac{+1}{(c - f_{dt} - f_{at})A}$	$c > f_{dt} + f_{at}$	a
(2)	2	$+r^2/y_t$	$\frac{-n}{(f_{dt} + f_{at} - c)A}$	$c < f_{dt} + f_{at}$	b
(9)	2'	$+r^2/y_t$	$\frac{-1}{(c_t - f'_{at} + f_{dt})A}$	$c_t > f'_{at} + f_{dt}$	c
(11)	2'	$+r^2/y_t$	$\frac{+n}{(f'_{at} - f_{dt} + c_t)A}$	$c_t < f'_{at} - f_{dt}$	d
(4)	4	$-r^2/y_b$	$\frac{+n}{(f_{db} + f_{ab} - c_t)A}$	$c_t < f_{db} + f_{ab}$	e
(10)	4'	$-r^2/y_b$	$\frac{+1}{(c - f'_{ab} + f_{db})A}$	$c > f'_{ab} - f_{db}$	f
(10)	4'	$-r^2/y_b$	$\frac{+1}{(f'_{ab} + f_{db} - c)A}$	$c > f'_{ab} - f_{db}$	g

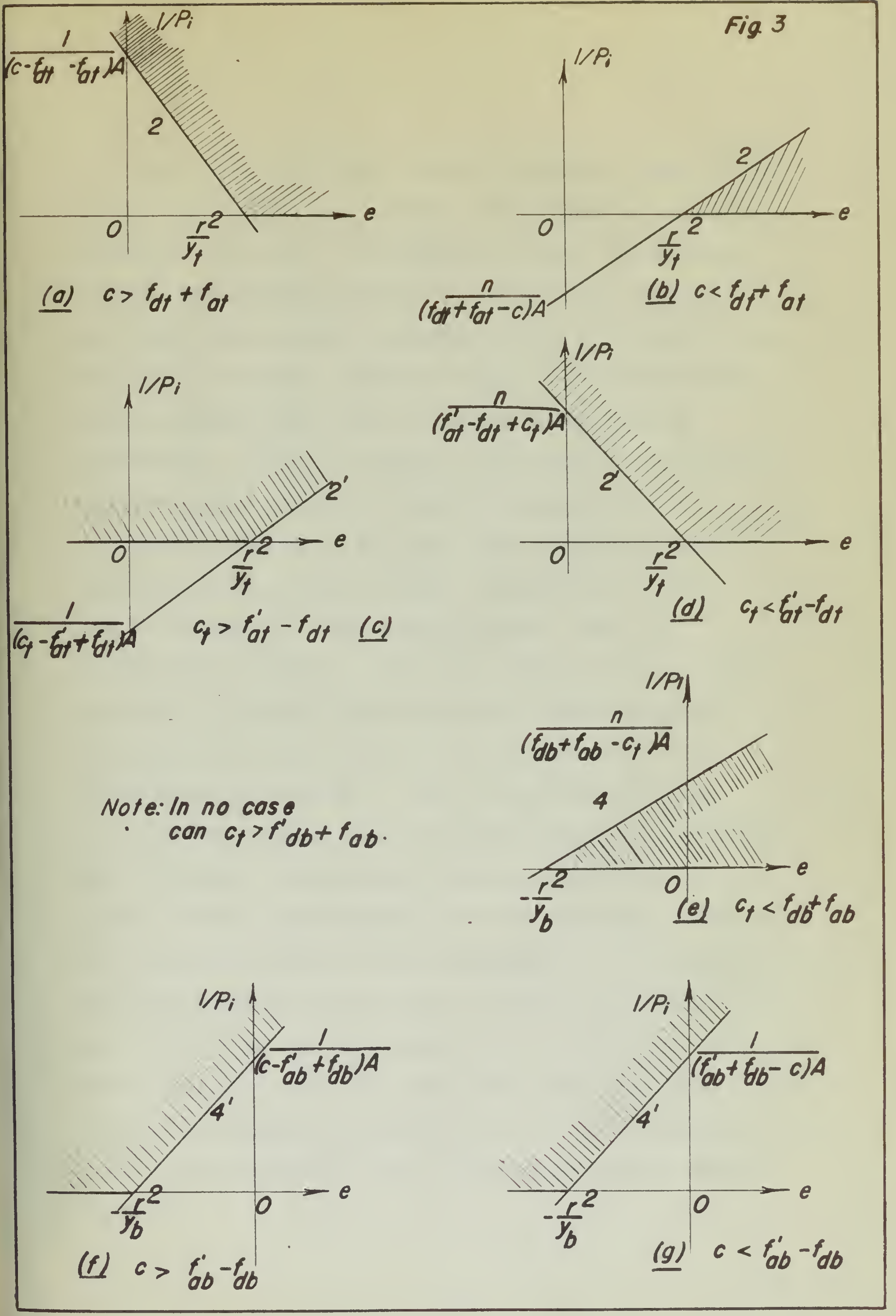


Fig 3

(a) $c > f_{dt} + f_{at}$

(b) $c < f_{dt} + f_{at}$

(c) $c_t > f'_{at} - f_{dt}$

(d) $c_t < f'_{at} - f_{dt}$

(e) $c_t < f_{db} + f_{ab}$

(f) $c > f'_{ab} - f_{db}$

(g) $c < f'_{ab} - f_{db}$

Note: In no case can $c_t > f'_{db} + f_{ab}$.

$1/P_i$
e
0
2
$2'$
4
$4'$
$\frac{r^2}{y_t}$
$-\frac{r^2}{y_b}$
$\frac{1}{(c - f_{dt} - f_{at})A}$
$\frac{n}{(f_{dt} + f_{at} - c)A}$
$\frac{1}{(c_t - f'_{at} + f_{dt})A}$
$\frac{n}{(f'_{at} - f_{dt} + c_t)A}$
$\frac{n}{(f_{db} + f_{ab} - c_t)A}$
$\frac{1}{(c - f'_{ab} + f_{db})A}$
$\frac{1}{(f'_{ab} + f_{db} - c)A}$

In Fig. 3 the shaded areas correspond to the values of (e) and ($1/P_i$) that satisfy the particular condition formula. To solve a particular section, the bending moments and stresses caused by the loads w_d, w_a and w'_a are found and then the ordinates for 2, 2', 4 and 4' are computed. The four condition lines are drawn and the area enclosed within the four lines satisfies the conditions. Similar diagrams of (e) versus ($1/P_i$) are drawn for the sections of maximum moments (for a continuous beam of three equal spans these are the midpoint of the exterior span, the interior support and the midpoint of the central span). This is illustrated in Fig. 4. Any line such as H-H' is drawn parallel to the (e) axis cutting all three areas and satisfies the conditions of the particular beam and its particular loading. Line H-H' gives the value of ($1/P_i$) from which the prestressing force is obtained. The intercepts of line H-H' with the areas give the limits of the eccentricities for the particular sections for which the diagrams are constructed. It is obvious that the further the line H-H' is from the (e) axis the smaller will be the prestressing force required. Thus it will be best to investigate this condition first and should the allowable eccentricities not be obtainable, other lines parallel to the (e) axis,but closer, should be tried.

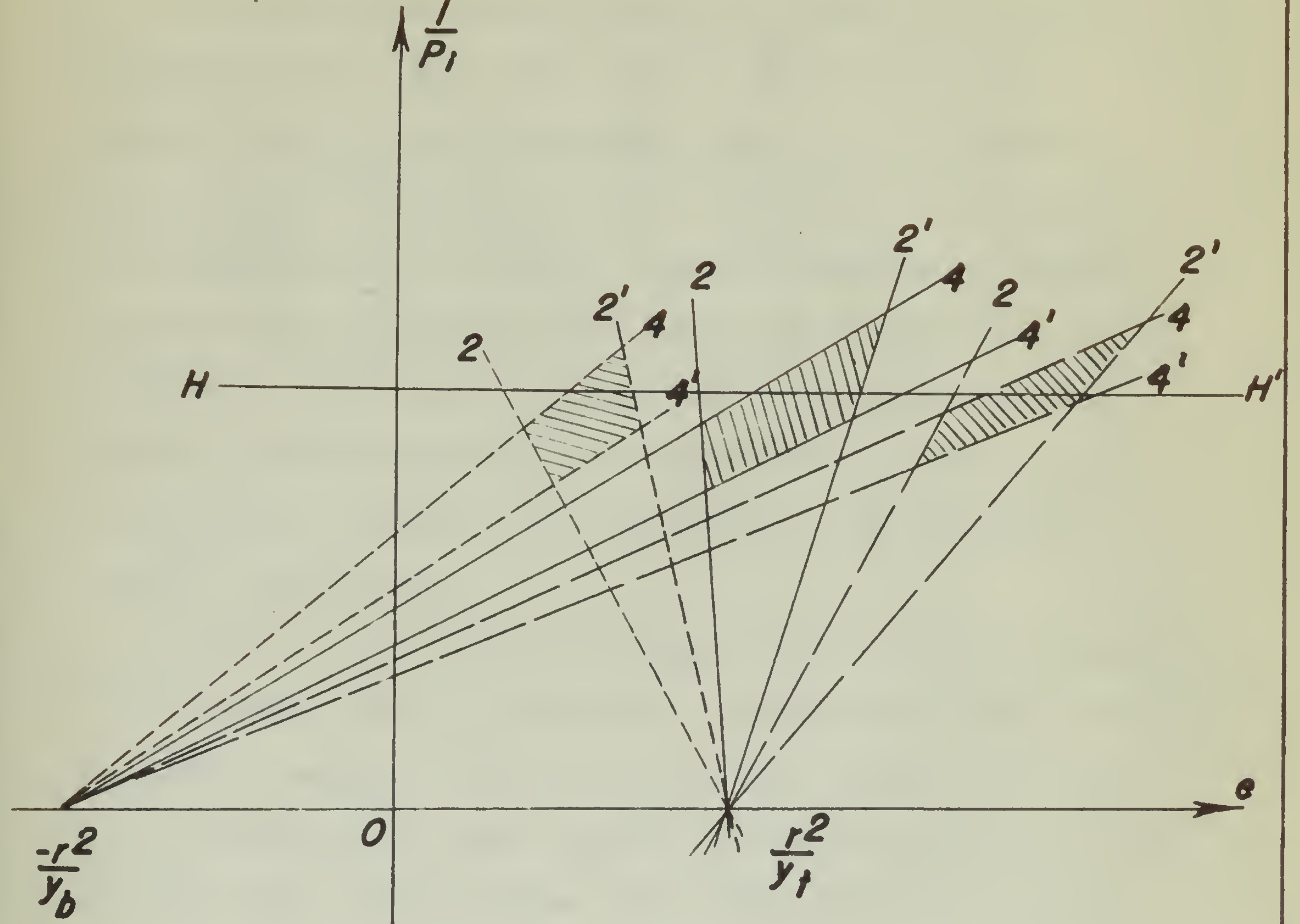


Fig. 4

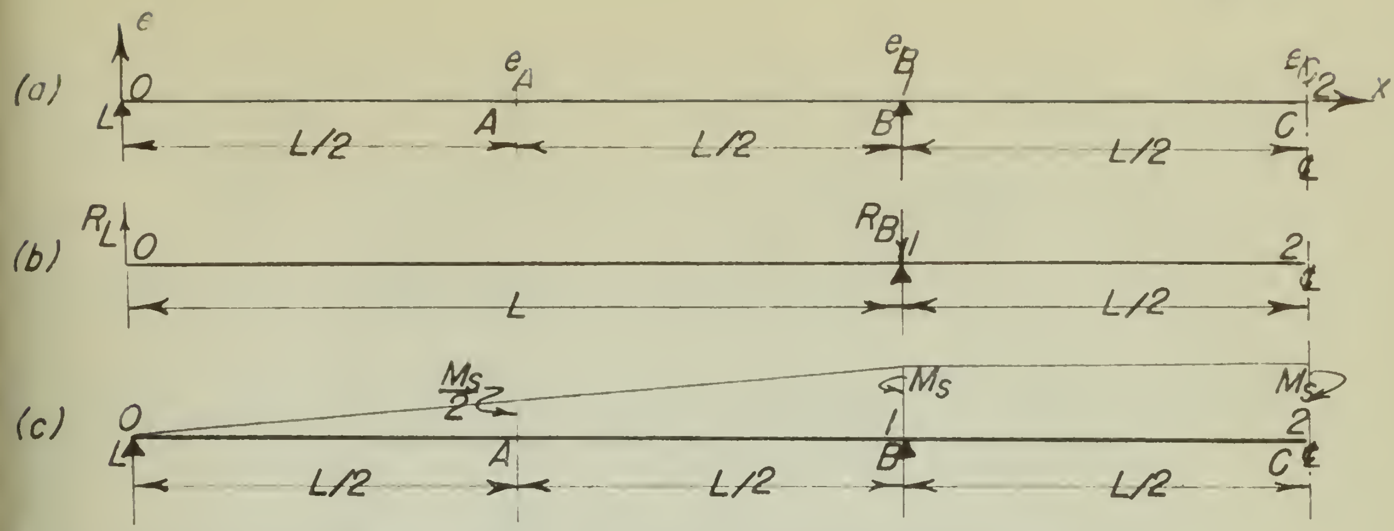
GRAPHICAL REPRESENTATION
of
CONDITION EQUATIONS
for
CONTINUOUS BEAMS
of
THREE EQUAL SPANS

SECONDARY BENDING MOMENT
AT
INTERIOR SUPPORT

The Secondary Bending Moment in terms of the prestressing force (P_i), the span length (L) and the eccentricity of the prestressing cable (e) has been derived from the Conjugate Beam Theory and the Principle of Superposition (the derivation is indicated by Magnel in his "Prestressed Concrete" with the intermediate steps omitted). The resulting expression for the Secondary Bending Moment is general and applicable to any assumed arrangement of cables and combination of prestressing forces. (see Fig. 5).

Magnel follows his recommendations for the eccentricity of the cable, that is, no cable eccentricity at the exterior supports, sagging of the cable in the middle of the end spans, humping over the interior supports, sagging at the middle of the central span, and being symmetrical about the midpoint of the central span. He accomplishes this by the use of a continuous cable as follows: in the exterior half of the ends span with a second degree parabola; in the interior half of the end spans with a fourth degree parabola and in the central span with another fourth degree parabola. Using the algebraic equations for the cable eccentricity and letter designations for the three

Fig. 5



THE SECONDARY MOMENT AT AN INTERIOR SUPPORT.

1. By Conjugate Beam,

$$(1) \quad \theta_1 = \theta_0 + \frac{1}{EI}\int_0^1 M\,dx \qquad \left(\text{Slope} = \text{area of } \frac{M}{EI} \text{ diagram}\right)$$

$$(2) \quad \Delta y_1 = \Delta y_0 + \theta_0(x_1 - x_0) + \frac{1}{EI}\int_0^1 M(x_1 - x)\,dx$$

$$\left(\text{Deflection} = \text{moment of area of } \frac{M}{EI} \text{ diagram}\right)$$

2. ASSUME Supports B & D removed; when prestressing force applied it produces a bending moment, $M (= P\cdot e)$, with a resulting deformation. Applying (1) & (2),

at midpoint C, $(1') \quad \theta_C = 0 = \theta_L + \frac{1}{EI}\int_0^c M\,dx \qquad \text{or} \quad \theta_L = -\frac{1}{EI}\int_0^c M\,dx. \ (1'')$

at support B, $(2') \quad \Delta y_B = 0 + \theta_L L + \frac{1}{EI}\int_0^B M(L - x)\,dx$

∴ substituting $M = P\cdot e$ and $(1'')$ in $(2')$,

$$(2'') \quad \Delta y_B = -\frac{L}{EI}\int_0^c Pe\,dx + \frac{1}{EI}\int_0^B P(L - x)\,e\,dx$$

$$= -\frac{PL}{EI}\int_0^c e\,dx + \frac{PL}{EI}\int_0^B e\,dx - \frac{P}{EI}\int_0 ex\,dx$$

$$= -\frac{P}{EI}\left[\int_0^B ex\,dx + L\int_B^c e\,dx\right] \quad (\text{upwards})$$

(deflection at B due to prestressing force, P).

3. Referring to beam in (b) above, apply load R_B l at B & D (R_B is the aforementioned force in discussion of problem of continuous beam).

from $\Delta x (\text{when } x \leqq a) = \frac{Px}{6EI}(3La - 3a^2 - x^2)$,

$$\Delta y_B = \frac{R_B L}{6EI}(3\cdot 3L\cdot L - 3L^2 - L^2) = +\frac{5R_B L^3}{6EI} \quad (\text{downwards})$$

controlling eccentricities (i.e., the middle of the end span, the interior support, and the middle of the central span), he solves for the value of the Secondary Bending Moment.

Using the concept of the equivalent eccentricity (the actual eccentricity plus an apparent eccentricity), he proceeds to set up a graph (see Fig. 6) which is entered with the limits of the equivalent eccentricities from the diagram of e' vs $1/P_i$ (see Fig. 4) and an arbitrarily chosen eccentricity for the interior support. Lines are drawn parallel to the basic construction lines and an area should be bounded by these line - any point of which satisfies the conditions of the problem. If the arbitrarily chosen value of the actual eccentricity of the interior support does not provide a satisfactory solution, others must be tried. If the greatest available actual eccentricity at the interior support does not provide a solution, the beam is too shallow for the condition of loading.

To eliminate the parabolic cable, the use of a graph with four construction lines, and the field construction of a parabolic cable, a simpler cable arrangement was investigated. A continuous parabolic cable, a continuous straight cable, a continuous straight cable with an additional cable over each interior support and discontinuous cables with various

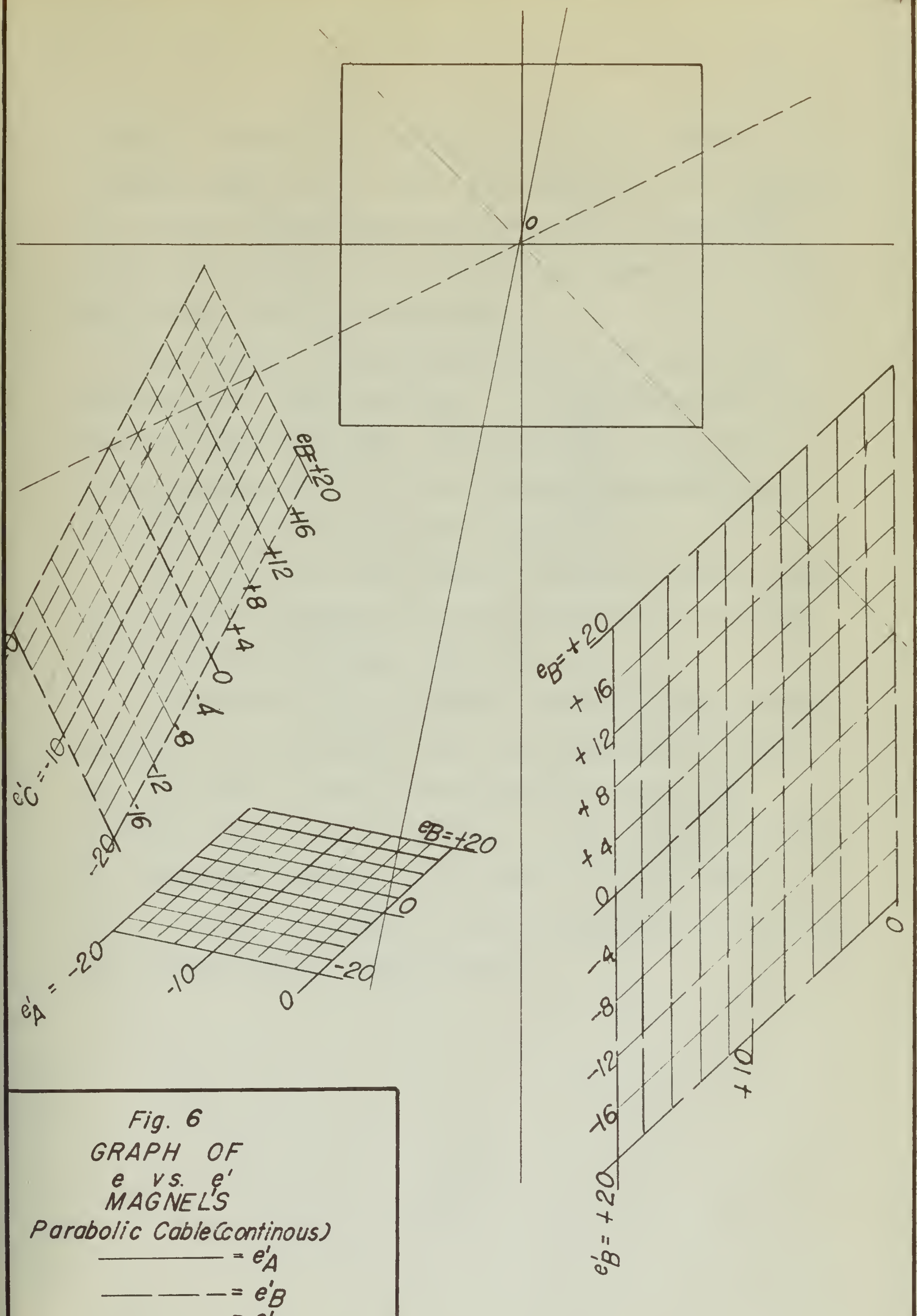


Fig. 6
GRAPH OF
e vs. e'
MAGNEL'S
Parabolic Cable(continous)

——————— = e'_A

——— ——— —= e'_B

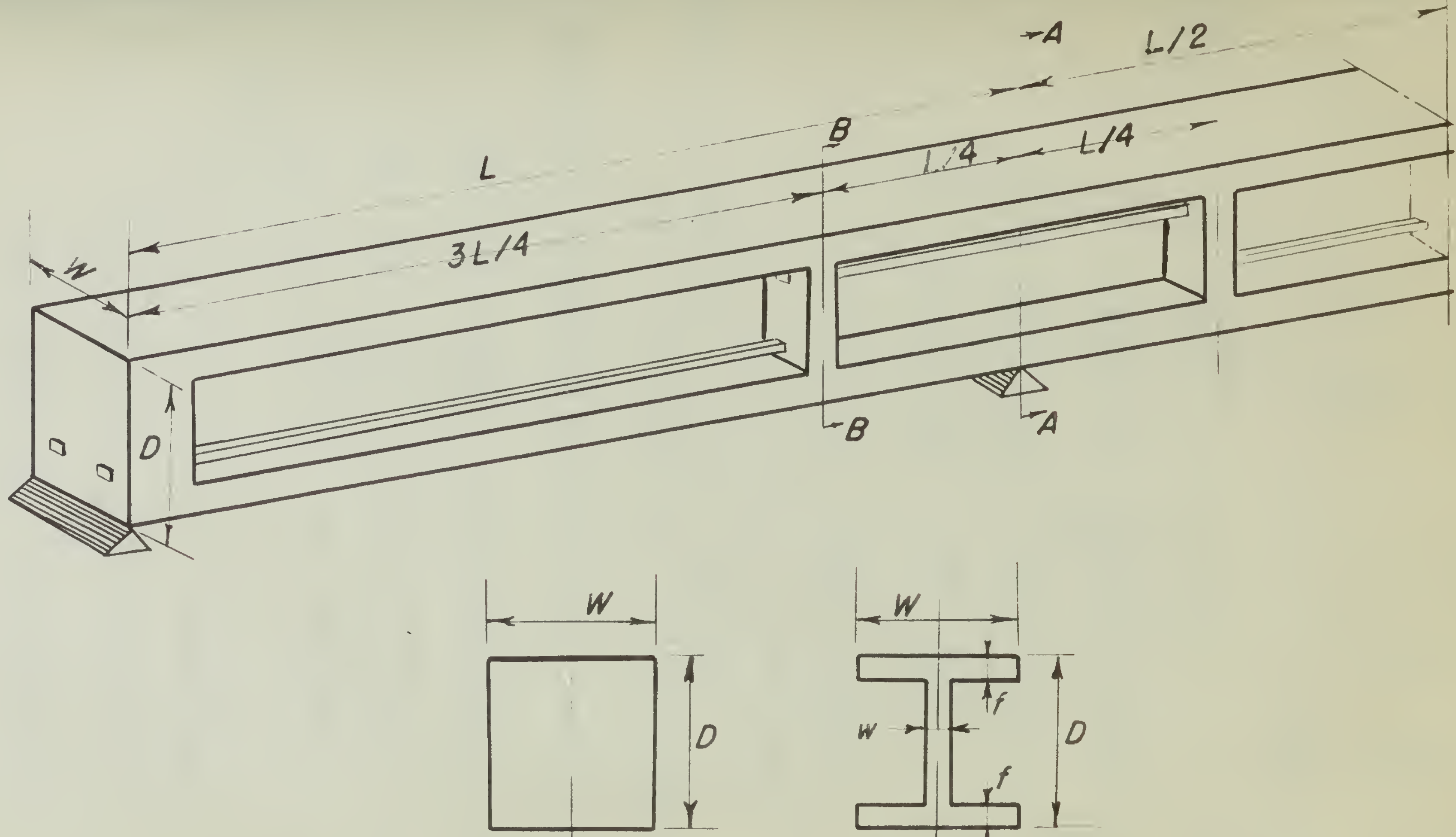
— — — — — — = e'_C

points of cut-off were investigated with no special results worthy of note (with the exception of the latter). The use of a discontinuous cable composed of straight cables with quarter point cut-offs was tried and is the one adopted for this presentation.

The cable is illustrated in Fig. 8 with all the distances and forces labelled. The derivation for the Secondary Bending Moment, the assumptions made and the final equations for the three equivalent eccentricities are for the cable arrangement indicated.

The curves for the equivalent eccentricities in terms of the actual eccentricities are determined by the three final defining equations. (see Fig. 9).

The procedure is as before - enter with the limiting equivalent eccentricities from the diagram of e'vs $1/P_i$ and the actual limiting eccentricity imposed by the physical setup of the chosn beam. A few quick trys and an acceptable solution can be found. If the actual eccentricities available are not satisfactory, the beam is too shallow for the loadings used.



Section A-A

Section B-B

Fig. 7
PROPOSED CABLE
ARRANGEMENT

Fig. 8

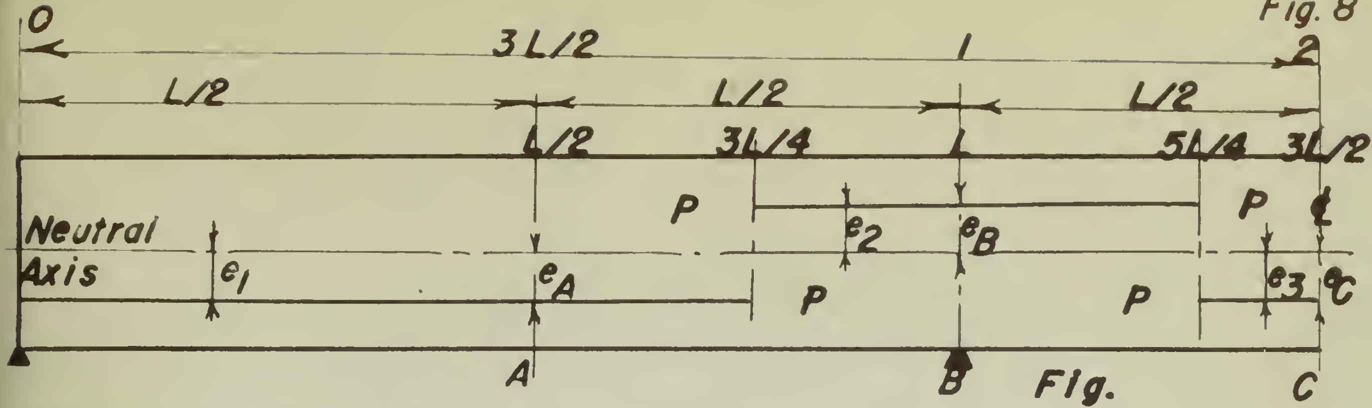


Fig.

ASSUME Same 'P' For All Cables.

1. From Eq. , $M_S = -\frac{6P}{5L^2}\left[\int_0^1 ex\,dx + L\int_1^2 e\,dx\right]$.

$$= -\frac{6P}{5L^2}\left[\int_0^{3L/4} e_1 x\,dx + \int_{3L/4}^{L} e_2 x\,dx + L\int_L^{5L/4} e_2\,dx + L\int_{5L/4}^{3L/2} e_3\,dx\right].$$

$$= -\frac{6P}{5L^2}\left[e_1 x^2/2\Big|_0^{3L/4} + e_2 x^2/2\Big|_{3L/4}^{L} + e_2 Lx\Big|_L^{5L/4} + e_3 Lx\Big|_{5L/4}^{3L/2}\right].$$

$$= -\frac{6P}{5L^2}\left[\frac{9e_1L^2}{32} + \frac{e_2L^2}{2} - \frac{9e_2L^2}{32} + \frac{e_2L^2}{4} + \frac{e_3L^2}{4}\right].$$

$$= -\frac{6P}{5}\left[\frac{9e_1}{32} + \frac{15e_2}{32} + \frac{8e_3}{32}\right].$$

$$= -P\left[0.3375\,e_1 + 0.5625\,e_2 + 0.3\,e_3\right].$$

ASSUME $e_1 = e_3 = e_A = e_C$; and $e_2 = e_B$.

2. $\therefore \quad M_S = -P\left[0.6375\,e_A + 0.5625\,e_B\right]$.

By definition, Equivalent Eccentricity $e' = e + \frac{M_S}{P}$.

3. $e'_A = e_A + \frac{M_S}{2P} = e_A - 0.31875\,e_A - 0.28125\,e_B$.

$e'_B = e_B + \frac{M_S}{P} = e_B - 0.6375\,e_A - 0.5625\,e_B$.

$e'_C = e_C + \frac{M_S}{P} = e_A - 0.6375\,e_A - 0.5625\,e_B$.

-or-

$e'_A = 0.68125\,e_A - 0.28125\,e_B$.

$e'_B = -0.6375\,e_A + 0.4375\,e_B$.

$e'_C = 0.3625\,e_A - 0.5625\,e_B$.

Draw curves for e'_A, e'_B, & e'_C in terms of e_A & e_B.

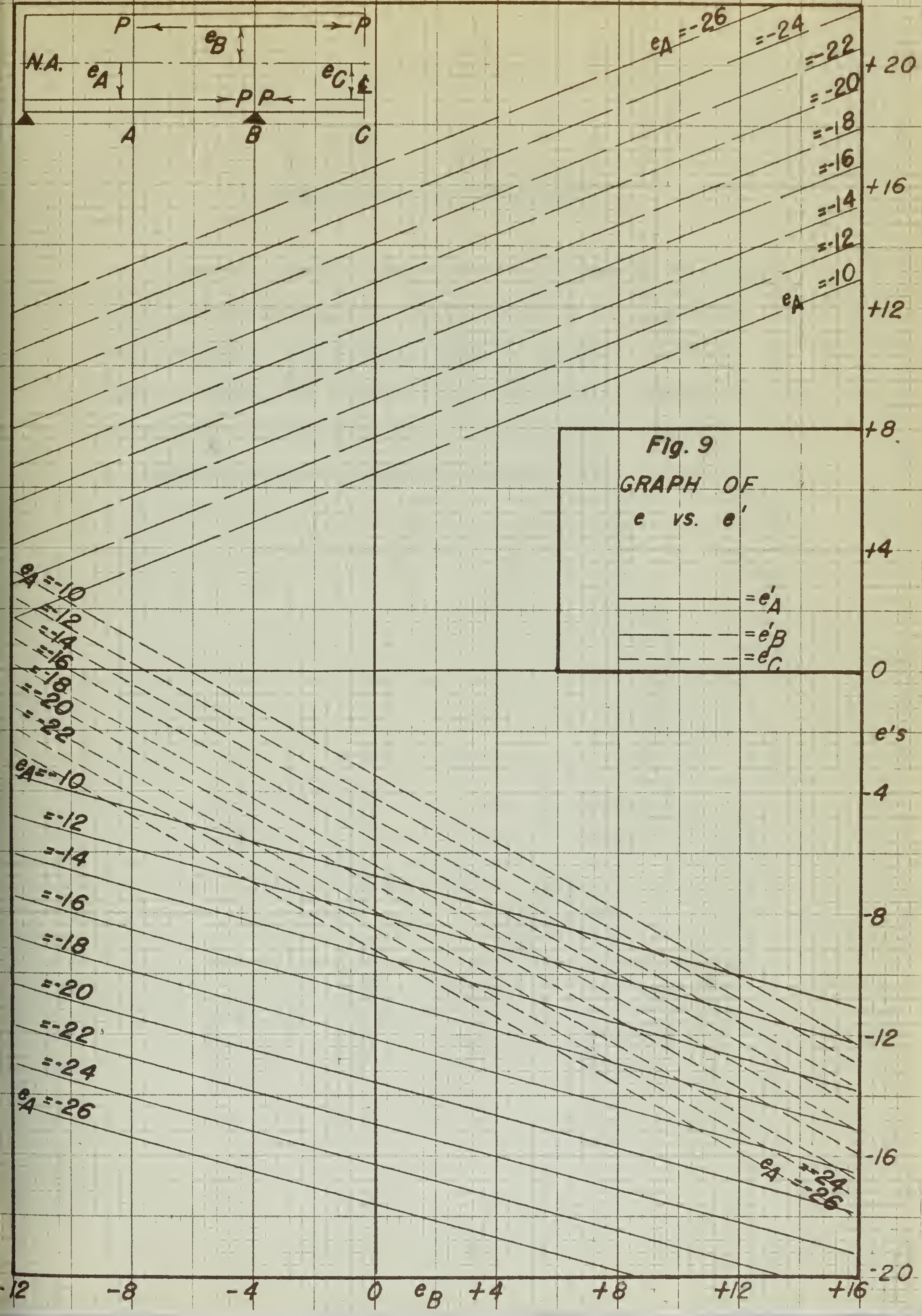


Fig. 9

GRAPH OF
e vs. e'

FOUNDATIONS AND ABUTMENTS

Since no special design or techniques is required for the construction of the foundations and abutments of prestressed reinforced concrete bridges, it is assumed that such abutments and foundations can be built with no difficulty. Therefore no analysis or design of these elements is made.

CONSTRUCTION PROCEDURE

No attempt is made to discuss the detailed construction procedure necessary for the practical application of this design. Such procedure would depend on the location and the engineering experience of the contractor. There is nothing in this design which requires the development of new methods of construction. The technique of construction already applied to existing prestressed concrete bridges could with small modifications be applied to the proposed design.

Since each girder in the designed bridge is 300 feet long and would weigh approximately 120 tons, a serious transportation and placement problem would result if the girders were cast at a distance from the site. For that reason, the girders would be advantageously cast on top of the piers, one or two at a time, using falsework to support the forms. This would require only enough falsework to support two or three girders. After prestressing the girders would be moved laterally into position. The design of the girders was predicated on this method of construction.

GENERAL FEATURES OF DESIGN

The bridge herein designed is a continuous span bridge consisting of three-one hundred (3-100') foot spans. The cross section of the bridge consists of seven 48" I-beam girders. On the top flange of the girders is a 3" wearing surface of concrete. This allows two lanes on a 26-foot roadway with two one-foot curbs, for an overall width of twenty-eight feet. (see Fig. 10).

Each girder is cast atop the piers and is post-tensioned with high strength steel wires by the Magnel-Blaton system before being placed in its proper position. The prestressing steel consists of straight wires covered with a metal sheath and grouted to form cables. These cables are placed on either side of the web of the I-beam and are anchored in specially designed rectangular blocks. (see Fig. 7). For additional protection the cable sheaths would be protected by a bituminous coating. The exterior side of the two outside girders would be covered with a thin shell of concrete giving protection to the otherw-se exposed cable sheaths. All the cables are below the neutral axis within the core of the section except over the interior supports where the cables are above the neutral axis to counteract the negative moment. These cables are not continuous, starting and stopping at predetermined points where the rectangular blocks are placed for anchoring. This discontinuity of the cables is the peculiar feature of this design and, as far as in known

SUMMARY OF PROPOSED DESIGN

The bridge herein designed is a continuous span bridge consisting of three-one hundred (3-100') foot spans. The cross section of the bridge consists of seven 48" I-beam girders. On the top flange of the girders is a 6" concrete surface of pavement. This allows two lanes of a 24-foot roadway with two one-foot curbs, for an overall width of twenty-eight feet. (see Fig. 10).

Each girder is made from one piece and is post-tensioned with high strength steel wires by the Magnel-Blaton system, being placed in its proper position. The post-tensioning steel consists of standard wires covered with a metal sheath and grouted in with cement. These cables are placed on either side of the web of the I-beam and are anchored in Magnel-Blaton anchorage blocks, (see Fig. 9). For additional protection the cable grouting would be protected by a waterproof coating. The exterior side of the two outside girders would be covered with a thin shell of concrete giving protection to the otherwise exposed cable sheaths. All the cables are below the neutral axis within the core of the section except over the interior supports where the cables are above the neutral axis to counteract the negative moment. These cables are not continuous, starting and stopping at pre-determined points where the particular cables are no longer necessary. This discontinuity of the cables is the peculiar feature of this design and, as far as is known

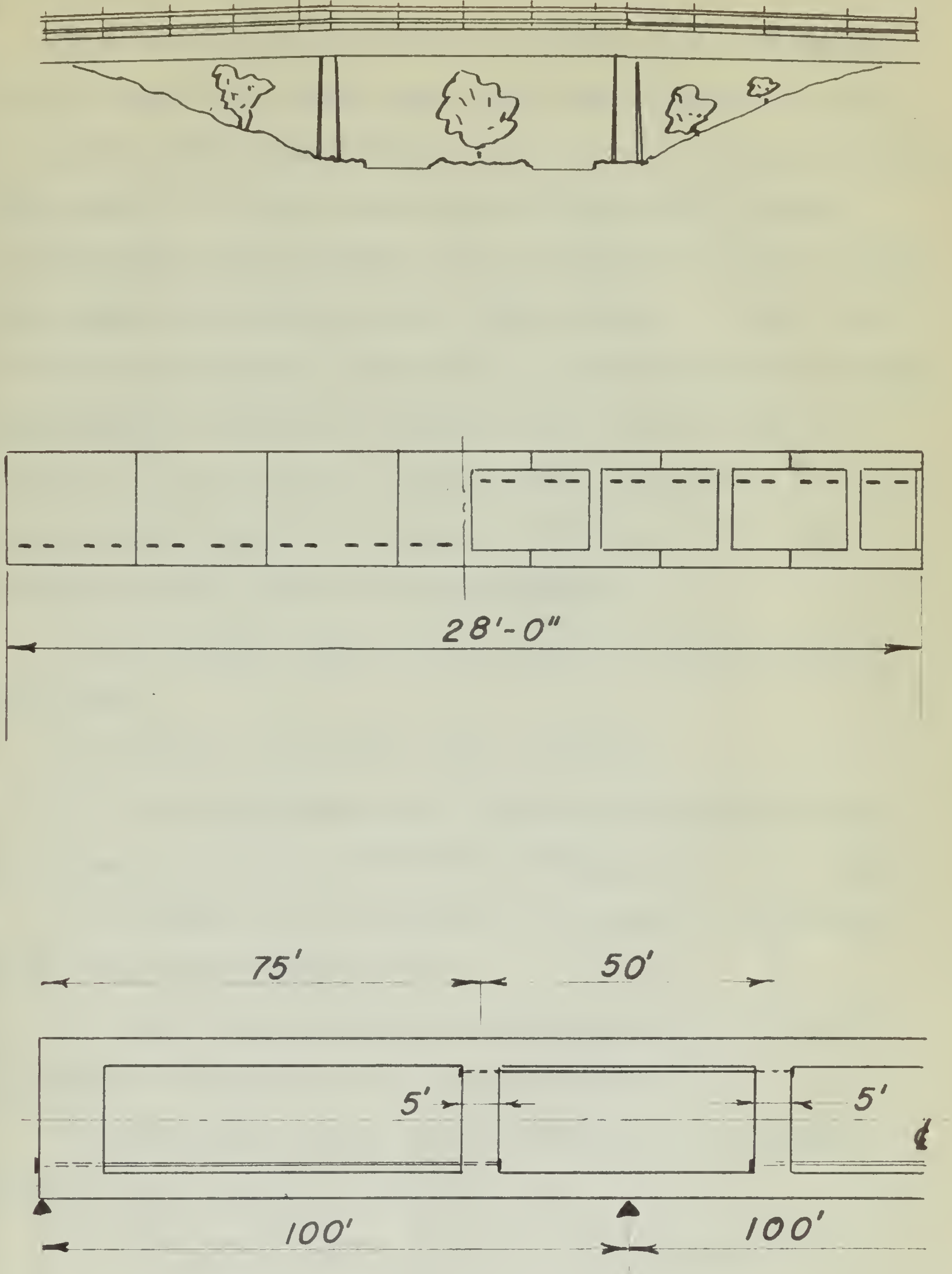


Fig. 10

GENERAL SKETCH, CROSS-SECTION
GIRDER DETAIL

by the authors, this particular type of construction has not been previously used.

Each of the seven girders is keyed to the adjoining one by shear keys on the upper and lower flanges. After the seven prestressed girders are in place on the abutments, steel wires in cables are passed transversely through the upper and lower flanges at a specified spacing, holes being left in the flanges for this purpose. These wires are then prestressed establishing a post-stressed condition transversely through the combined and lower flanges. Because of the stress so established, each girder does not act individually but assumes the status of a monolithic structure with the other girders.

The following steps are the basis of the procedure for design:

(1) Assume a concrete cross section.

(2) For dimensions used, find the limiting values of the equivalent eccentricities (diagram of e' vs $1/P_i$).

(3) Express the variation of the cable eccentricity as an algebraical expression.

(4) Find the equivalent eccentricities in terms of the actual eccentricities. (diagram of e vs e').

(5) Determine actual eccentricities by satisfying diagrams of e' vs $1/P_i$ and e vs e'.

(6) From the diagram of e' vs $1/P_i$ determine the value of the prestressing force.

(7) Compute Secondary Bending Moment and its induced stresses.

(8) Combine stresses from loads at time of prestress, stresses from superimposed loads, stresses due to the prestressing and stresses due to Secondary Bending Moment in all possible combinations (at the time of prestress, after prestress with superimposed loads, and after an elapsed time) to see if stresses conform to the allowable compressive and tensile strengths.

(9) The remainder of the design is for details - such as the check on the shear, the design of the shear key, the design of the anchorage, the design of the transverse steel and so forth.

SPECIFICATIONS

Since prestressed concrete construction is relatively new in this country, no standard set of specifications have been agreed upon. Where possible the specifications set forth in the current American Concrete Institute Standards were adhered to. For those instances, in which it was believed that the published specifications were not applicable, values as recommended in 'publications' by Gustav Magnel, P. W. Abeles and various other authorities were used.

With regard to loading, the standard specifications for Highway Bridges, fifth edition, 1949, as adopted by the American Association of State Highway Officials was used.

The following specifications were followed in the design:

Live Load

H-20-S16-44 (AASHO Standard Specifications for Highway Bridges)

Impact Factor

$$I = \frac{50}{L + 125}$$

L = Length between supports in feet

Dead Load

Two lanes on a 26 foot roadway.

Two one-foot curbs equal to 300 lbs. per foot of bridge

Three-inch road wearing surface equal to 150 lbs. per foot of girder

Maximum Deflection Allowable

Live load plus impact - $\frac{1}{300}$ x span

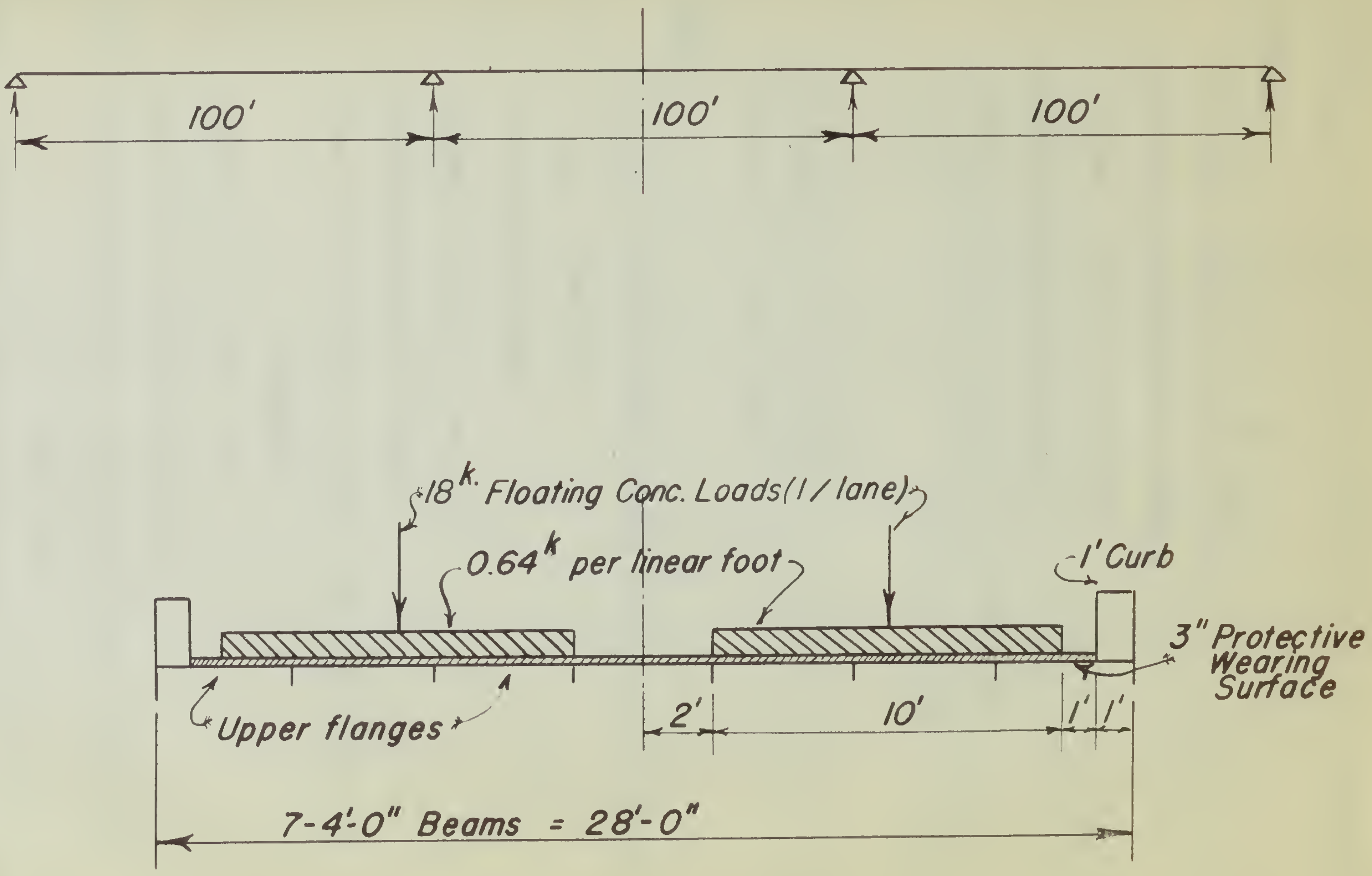


Fig. 11

SYSTEM OF LOADING

PROPERTIES OF MATERIALS

The materials used in this design have the following properties:

CONCRETE

Ultimate compressive strength	6000 psi
Allowable compressive stress	2000 psi
Ultimate tensile strength	700 psi
Allowable tensile stress	0 psi
Allowable shear stress (no reinforcement)	120 psi
(stirrups alone as web reinforcement)	360 psi
Allowable bearing stress	3500 psi
Modulus of elasticity	3,000,000 psi

STEEL

Special Roebling Acid Steel Prestressed Concrete Wire

Ultimate strength	240,000 psi
0.7% Elongation at	180,000 psi
Minimum ultimate elongation	4%
Allowable design stress	120,000 psi
Sress after creep of steel and shrinkage of concrete	102,000 psi
Allowable load per wire	7,200 lbs (0.276" D)
Allowable tensioning stress	135,000 psi

web Web Reinforcement and Anchor Block Wire Cage

Design stress	20,000 psi

NOTES: The above concrete and steel stress were assumed after having investigated those set forth by various authorities as follows:

Concrete

a) Magnel - "Prestressed Concrete"
- Ultimate compressive strength of 8800 psi(7500 psi at time of prestressing)
- Working compressive stress of 1/4 to 1/3 of ultimate
- Bearing stress of 1.75 times working compressive stress
- Tensile stress of 1/10 of compressive stress (2/10 if reinforcing provided)
- Shear stress of 1/10 of compressive stress

b) Portland Cement Association - Modern Developments in Reinforced Concrete, No. 25, "Design of Prestressed Concrete"
- Ultimate compressive strength of 5000 psi
- Working compressive stress of 2000 psi
- No working tensile stress under design loads
- Allowable tensile stress extreme fibre for check on cracking of 700 psi

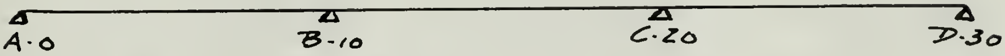
c) ACI Standards - 1946
- Shear stress as 2/100 ultimate compressive strength (no reinforcing provided)
- Shear Sress as 8/100 with web reinforcement of stirrups only

Steel

Roebling Wire Company - Specifications for Special Roebling Acid Steel Prestressed-Concrete Wire

MOMENT DISTRIBUTION

Continuous beam of 3-100' spans.
Unit loadings at each 10' interval.



Point Loaded	A	B		C		D
	AB	BA	BC	CB	CD	DC
	1	3/4	3/4	1	1	1
	0	0.428	0.572	0.572	0.428	0
1	+8.10	-0.90				
	-8.10	-4.05				
		+2.12	+2.83	+1.42		
			-0.40	-0.81	-0.61	
		+0.17	+0.23	+0.12		
				-0.07	-0.05	
	0	-2.66	+2.66	+0.66	-0.66	0
2	+12.80	-3.20				
	-12.80	-6.40				
		+4.11	+5.49	+2.74		
			-0.78	-1.57	-1.17	
		+0.34	+0.44	+0.22		
				-0.13	-0.09	
	0	-5.16	+5.15	+1.26	-1.26	0
3	+14.70	-6.30				
	-14.70	-7.35				
		+5.85	+7.80	+3.90		
			-1.12	-2.23	-1.67	
		+0.48	+0.64	+0.32		
				-0.18	-0.14	
	0	-7.32	+7.32	+1.81	-1.81	0
4	+14.40	-9.60				
	-14.40	-7.20				
		+7.19	+9.61	+4.80		
			-1.37	-2.74	-2.06	
		+0.59	+0.78	+0.39		
			-0.11	-0.22	-0.17	
		+0.05	+0.06			
	0	-8.97	+8.97	+2.23	-2.23	0
5	+12.50	-12.50				
	-12.50	- 6.25				
		+8.05	+10.70	+5.35		
			-1.53	-3.06	-2.29	
		+0.65	+0.88	+0.44		
			-0.12	-0.25	-0.19	
		+0.05	+0.07			
	0	-10.00	+10.00	+2.48	-2.48	0

Point Loaded	A	B		C		D
	AB	BA	BC	CB	CD	DC
6	+9.60	-14.40				
	-9.60	- 4.80				
		+8.20	+11.00	+5.50		
			-1.58	-3.15	-2.35	
		+0.68	+0.90	+0.45		
			-0.12	-0.25	-0.20	
		+0.05	+0.07			
	0	-10.27	+10.27	+2.55	-2.55	0
7	+6.30	-14.70				
	-6.30	-3.15				
		+7.65	+10.20	+5.10		
			-1.46	-2.92	-2.18	
		+0.62	+0.84	+0.42		
			-0.12	-0.24	-0.18	
		+0.05	+0.07			
	0	-9.53	+9.53	+2.36	-2.36	0
8	+3.20	-12.80				
	-3.20	-1.60				
		+6.16	+8.24	+4.12		
			-1.18	-2.36	-1.76	
		+0.50	+0.68	+0.34		
			-0.10	-0.19	-0.15	
		+0.04	+0.06			
	0	-7.70	+7.70	+1.91	-1.91	0
9	+0.90	-8.10				
	-0.90	-0.45				
		+3.66	+4.89	+2.44		
			-0.70	-1.40	-1.04	
		+0.30	+0.40	+0.20		
				-0.09	-0.11	
	0	-4.59	+4.59	+1.15	-1.15	
11			+8.10	-0.90		
		-3.47	-4.63	-2.32		
			+0.92	+1.84	+1.38	
		-0.39	-0.53	-0.26		
			+0.08	+0.15	+0.11	
		-0.03	-0.05			
	0	-3.89	+3.89	-1.49	+1.49	
12			+12.80	-3.20		
		-5.48	-7.32	-3.66		
			+1.96	+3.92	+2.94	
		-0.84	-1.12	-0.56		
			+0.16	+0.32	+0.24	
		-0.07	-0.09			
	0	-6.39	+6.39	-3.18	+3.18	

Point Loaded	A	B		C		D
	AB	BA	BC	CB	CD	DC
13			+14.70	-6.30		
		-6.29	-8.41	-4.20		
			+3.00	+6.01	+4.49	
		-1.28	-1.72	-0.86		
			+0.24	+0.49	+0.37	
		-0.10	-0.14	-0.07		
				+0.04	+0.03	
	0	-7.67	+7.67	-4.89	+4.89	0
14			+14.40	-9.60		
		-6.16	-8.24	-4.12		
			+3.92	+7.85	+5.87	
		-1.68	-2.24	-1.12		
			+0.32	+0.64	+0.48	
		-0.14	-0.18	-0.09		
				+0.05	+0.04	
	0	-7.98	+7.98	-6.39	+6.39	0
15			+12.50	-12.50		
		-5.35	-7.15	-3.58		
			+4.60	+9.20	+6.88	
		-1.97	-2.63	-1.32		
			+0.38	+0.75	+0.57	
		-0.16	-0.22	-0.11		
				+0.06	+0.05	
	0	-7.48	+7.48	-7.50	+7.50	0

COMPILATION OF RESULTS OF MOMENT DISTRIBUTIONS FOR UNIT LOAD AT 10' INTERVAL

Point Loaded	AB	BA	BC	CB	CD	DC
A	0	0	0	0	0	0
1	0	-2.66	+2.66	+0.66	-0.66	0
2	0	-5.16	+5.16	+1.26	-1.26	0
3	0	-7.32	+7.32	+1.81	-1.81	0
4	0	-9.06	+9.06	+2.25	-2.25	0
5	0	-10.00	+10.00	+2.48	-2.48	0
6	0	-10.27	+10.27	+2.55	-2.55	0
7	0	-9.53	+9.53	+2.36	-2.36	0
8	0	-7.70	+7.70	+1.91	-1.91	0
9	0	-4.59	+4.59	+1.15	-1.15	0
B-10	0	0	0	0	0	0
11	0	-3.89	+3.89	-1.49	+1.49	0
12	0	-6.39	+6.39	-3.18	+3.18	0
13	0	-7.67	+7.67	-4.89	+4.89	0
14	0	-7.98	+7.98	-6.39	+6.39	0
15	0	-7.48	+7.48	-7.50	+7.50	0
16	0	-6.39	+6.39	-7.98	+7.98	0
17	0	-4.89	+4.89	-7.67	+7.67	0
18	0	-3.18	+3.18	-6.39	+6.39	0
19	0	-1.49	+1.49	-3.89	+3.89	0
C-20	0	0	0	0	0	0
21	0	+1.15	-1.15	-4.59	+4.59	0
22	0	+1.91	-1.91	-7.70	+7.70	0
23	0	+2.36	-2.36	-9.53	+9.53	0
24	0	+2.55	-2.55	-10.27	+10.27	0
25	0	+2.48	-2.48	-10.00	+10.00	0
26	0	+2.25	-2.25	-9.06	+9.06	0
27	0	+1.81	-1.81	-7.32	+7.32	0
28	0	+1.26	-1.26	-5.16	+5.16	0
29	0	+0.66	-0.66	-2.66	+2.66	0
D-30	0	0	0	0	0	0

REACTIONS

INFLUENCE LINE ORDINATES

Point Loaded	Reaction Left(L)	B'	+	Reaction B"	=	B
1	+0.873	+0.127		+0.033		+0.160
2	+0.748	+0.252		+0.064		+0.316
3	+0.627	+0.373		+0.091		+0.464
4	+0.509	+0.491		+0.113		+0.604
5	+0.400	+0.600		+0.125		+0.725
6	+0.297	+0.703		+0.128		+0.831
7	+0.205	+0.795		+0.119		+0.914
8	+0.123	+0.877		+0.096		+0.973
9	+0.054	+0.946		+0.057		+1.003
10	0.000	+1.000		0.000		+1.000
11	-0.039	+0.039		+0.924		+0.963
12	-0.064	+0.064		+0.832		+0.896
13	-0.077	+0.077		+0.728		+0.805
14	-0.080	+0.080		+0.616		+0.696
15	-0.075	+0.075		+0.500		+0.575
16	-0.064	+0.064		+0.384		+0.448
17	-0.048	+0.048		+0.272		+0.320
18	-0.032	+0.032		+0.168		+0.200
19	-0.015	+0.015		+0.076		+0.091
20	0.000	0.000		0.000		0.000
21	+0.011	-0.011		-0.057		-0.068
22	+0.019	-0.019		-0.096		-0.115
23	+0.024	-0.024		-0.119		-0.143
24	+0.026	-0.026		-0.128		-0.154
25	+0.025	-0.025		-0.125		-0.150
26	+0.022	-0.022		-0.113		-0.135
27	+0.018	-0.018		-0.091		-0.109
28	+0.013	-0.013		-0.064		-0.077
29	+0.006	-0.006		-0.033		-0.039
30	0.000	0.000		0.000		0.000

MOMENTS

INFLUENCE LINE ORDINATES

Point Loaded	M-1	M-2	M-3	M-4	M-5
1	+8.73	+7.46	+6.19	+4.92	+3.65
2	+7.48	+14.96	+12.44	+9.92	+7.40
3	+6.27	+12.54	+18.81	+15.08	+11.35
4	+5.09	+10.18	+15.27	+20.36	+15.45
5	+4.00	+8.00	+12.00	+16.00	+20.00
6	+2.97	+5.94	+8.91	+11.88	+14.85
7	+2.05	+4.10	+6.15	+8.20	+10.25
8	+1.23	+2.46	+3.69	+4.92	+6.15
9	+0.54	+1.08	+1.62	+2.16	+2.70
10	0.00	0.00	0.00	0.00	0.00
11	-0.39	-0.78	-1.17	-1.56	-1.95
12	-0.64	-1.28	-1.92	-2.56	-3.20
13	-0.77	-1.54	-2.31	-3.08	-3.85
14	-0.80	-1.60	-2.40	-3.20	-4.00
15	-0.75	-1.50	-2.25	-3.00	-3.75
16	-0.64	-1.28	-1.92	-2.56	-3.20
17	-0.48	-0.96	-1.44	-1.92	-2.40
18	-0.32	-0.64	-0.96	-1.28	-1.60
19	-0.15	-0.30	-0.45	-0.60	-0.75
20	0.00	0.00	0.00	0.00	0.00
21	+0.11	+0.22	+0.33	+0.44	+0.55
22	+0.19	+0.38	+0.57	+0.76	+0.95
23	+0.24	+0.48	+0.72	+0.96	+1.20
24	+0.26	+0.52	+0.78	+1.04	+1.30
25	+0.25	+0.50	+0.75	+1.00	+1.25
26	+0.22	+0.44	+0.66	+0.88	+1.10
27	+0.18	+0.36	+0.54	+0.72	+0.90
28	+0.13	+0.26	+0.39	+0.53	+0.65
29	+0.06	+0.12	+0.18	+0.24	+0.30
30	0.00	0.00	0.00	0.00	0.00

AREA OF INFLUENCE LINES

0-10	+399.7	+666.0	+865.7	+932.0	+931.7
10-20	-49.9	-99.7	-149.6	-199.4	-249.3
20-30	+16.5	+33.1	+49.6	+66.1	+82.7

Point Loaded	M-6	M-7	M-8	M-9	M-10
1	+2.38	+1.11	-0.16	-1.43	-2.70
2	+4.88	+2.36	-0.16	-2.68	-5.20
3	+7.62	+3.89	+0.15	-3.57	-7.30
4	+10.54	+5.63	+0.72	-4.19	-9.10
5	+14.00	+8.00	+2.00	-4.00	-10.00
6	+17.82	+10.79	+3.76	-3.27	-10.30
7	+12.30	+14.35	+6.40	-1.55	-9.50
8	+7.38	+ 8.61	+9.84	+1.07	-7.30
9	+3.24	+3.78	+4.32	+4.86	-4.60
10	0.00	0.00	0.00	0.00	0.00
11	-2.34	-2.73	-3.12	-3.51	-3.90
12	-3.84	-4.48	-5.12	-5.76	-6.40
13	-4.62	-5.39	-6.16	-6.93	-7.70
14	-4.80	-5.60	-6.40	-7.20	-8.00
15	-4.50	-5.25	-6.00	-6.75	-7.50
16	-3.84	-4.48	-5.12	-5.76	-6.40
17	-2.88	-3.36	-3.84	-4.32	-4.80
18	-1.92	-2.24	-2.56	-2.88	-3.20
19	-0.90	-1.05	-1.20	-1.35	-1.50
20	0.00	0.00	0.00	0.00	0.00
21	+0.66	+0.77	+0.88	+0.99	+1.10
22	+1.14	+1.33	+1.52	+1.71	+1.90
23	+1.44	+1.68	+1.92	+2.16	+2.40
24	+1.56	+1.82	+2.08	+2.34	+2.60
25	+1.50	+1.75	+2.00	+2.25	+2.50
26	+1.32	+1.54	+1.76	+1.98	+2.20
27	+1.08	+1.26	+1.44	+1.62	+1.80
28	+0.78	+0.91	+1.04	+1.17	+1.30
29	+0.36	+0.42	+0.48	+0.54	+0.60
30	0,00	0.00	0.00	0.00	0.00

AREA OF INFLUENCE LINES

0-10	+798.0	+597.7	+267.2(-2.0)	-208.3 (+56.7)	-669.3
10-20	-299.1	-349.0	-398.9	-448.7	-498.5
20-30	+99.2	+115.7	+132.3	+148.8	+165.9

Point Loaded	M-11	M-12	M-13	M-14	M-15
1	-2.37	-2.04	-1.71	-1.38	-1.05
2	-4.56	-3.92	-3.28	-2.64	-2.00
3	-6.39	-5.48	-4.57	-3.66	-2.75
4	-7.97	-6.84	-5.71	-4.58	-3.45
5	-8.75	-7.50	-6.25	-5.00	-3.75
6	-9.02	-7.74	-6.46	-5.18	-3.90
7	-8.31	-7.12	-5.93	-4.74	-3.55
8	-6.74	-5.78	-4.72	-3.86	-2.90
9	-4.07	-3.46	-2.89	-2.32	-1.75
10	0.00	0.00	0.00	0.00	0.00
11	+5.34	+4.58	+3.82	+3.06	+2.30
12	+1.92	+10.24	+8.56	+6.88	+5.20
13	-0.42	+6.86	+14.14	+11.44	+8.70
14	-1.84	+4.32	+10.48	+16.64	+12.80
15	-2.50	+2.50	+7.50	+12.50	+17.50
16	-2.56	+1.28	+5.12	+8.96	+12.80
17	-2.08	+0.64	+3.36	+6.08	+8.80
18	-1.52	+0.16	+1.84	+3.52	+5.20
19	-0.74	+0.02	+0.78	+1.54	+2.30
20	0.00	0.00	0.00	0.00	0.00
21	+0.53	-0.04	-0.61	-1.18	-1.75
22	+0.94	-0.02	-0.98	-1.94	-2.90
23	+1.21	+0.02	-1.17	-2.36	-3.55
24	+1.32	+0.04	-1.24	-2.52	-3.80
25	+1.25	0.00	-1.25	-2.50	-3.75
26	+1.07	-0.06	-1.19	-2.32	-3.45
27	+0.90	0.00	-0.90	-1.80	-2.70
28	+0.66	+0.02	-0.64	-1.26	-1.90
29	+0.27	-0.06	-0.39	-0.72	-1.05
30	0.00	0.00	0.00	0.00	0.00

AREA OF INFLUENCE LINES

0-10	-587.1	-503.2	-419.1	-336.4	-253.0
10-20	-116.0(+67.8)	+301.3	+568.0	+701.6	+768.0
20-30	+82.1	--	-84.6	-194.4	-253.0

DESIGN MOMENTS

Designation of Critical Points

A = midpoint of first span
B = first interior support
C = midpoint of central span

LIVE LOADS -

Section A
Positive
Unif. = 1014.4 X 1.28 = 1300
Conc. = 20 X 36 = 720
Total (+ impact)= 2020 X 1.22 = +2460 fk
Negative
Unif. = 249.3 X 1.28 = 319
Conc. = 4 X 36 = 144
Total (+ impact)= 463 X 1.22 = - 566 fk

Section B
Positive
Unif. = 165.9 X 1.28 = 212
Conc. = 2.55 X 36 = 92
Total (+ impact)= 304 X 1.22 = + 372 fk
Negative
Unif. = 1167.8 X 1.28 = 1491
Conc. = 18.27 X 36 = 660
Total (+impact)= 2151 X 1.22 = -2666 fk

Section C
Positive
Unif. = 768 X 1.28 = 984
Conc. = 17.5 X 36 = 630
Total (+ impact)=1614 X 1.22 = +1971 fk
Negative
Unif. = 507 X 1.28 = 650
Conc. = 3.9 X 36 = 141
Total (+ impact)= 791 X 1.22 = - 965 fk

CURBS AND RAILINGS (300 ppf - effective after prestress)

Section A = (0.3)(1014.4 - 249.3) = (0.3)(765.1)= + 229.3 fk
Section B = (0.3)(1167.8 - 165.9) = (0.3)(1001.9)=- 300.6 fk
Section C = (0.3)(768 - 507.0) = (0.3)(261) = + 78.6 fk

WEARING SURFACE (150 ppf - per girder - effective after prestres

Section A = (0.15)(765.1)(7) = 793.6 fk
Section B = (0.15)(1001.9)(7) = 1052.0 fk
Section C = (0.15)(261)(7) = 275.1 fk

DEAD LOADS

Section A = (0.15)(765.1)(A) = 114.8 A fk Section B = (0.15)(1001.9)(A)= 150.3 A fk Section C = (0.15)(261)(A) = 39.3 A fk	(A= Area of beam cross-section in sq.ft.)

TOTAL DESIGN MOMENTS PER 4' GIRDER

	Positive	Negative
Midpoint First Span (A)		
Dead Load	114.8A fk	-
Load after Prestress	497.6 fk	-
First Interior Support (B)		
Dead Load	-	150.3A fk
Load after Prestress	-	574.1 fk
Midpoint Central Span (C)		
Dead Load	39.3A fk	-
Load after Prestress	322.1 fk	87.3 fk

LOADING FOR MAXIMUM SHEAR

The loading for maximum shear is similar to a simple beam in that the critical shears will occur at the supports in a continuous beam. With three spans the maximum shear occurs at the first interior support. Since stirrups are required due to the use of 0.06 f'_c it is necessary to compute the stress not only at the supports but also at the points to the right and left of the first interior support. For computing these shears the specifications for maximum shear as stated in the AASHO Standard Specifications will again be followed. This requires a lane loading of 640 lb. per foot plus a 26,000 lb. concentration placed in such manner as to produce maximum stress. The live load is also required to be increased for impact.

To determine the maximum shear at the required points under the moving load, influence lines are again resorted to. Determining the areas of these influence lines and combining these with the required loads gives the maximum shear at the section under consideration.

INFLUENCE LINE ORDINATES FOR SHEAR

Point Loaded	V-A	V-B	V-8	V-9	V-11
0	+1.000	0.00	0.00	0.00	0.00
1	+.873	+.160	+.127	+.127	-.03
2	+.748	+.316	+.252	+.252	-.064
3	+.627	+.464	+.373	+.373	-.091
4	+.509	+.604	+.491	+.491	-.113
5	+.400	+.725	+.600	+.600	-.125
6	+.297	+.831	+.703	+.703	-.128
7	+.205	+.914	+.795	+.795	-.119
8	+.123	+.973	+.877(-123)	+.877	-.096
9	+.054	+1.003	+.054	+.946(-.054)	-.057
10	0.00	+1.000	0.000	0.000	0.000
11	-.039	+.963	+.039	+.039	+.076(-.924)
12	-.064	+.896	+.064	+.064	-.832
13	-.077	+.805	+.077	+.077	-.728
14	-.080	+.696	+.080	+.080	-.616
15	-.075	+.575	+.075	+.075	-.500
16	-.064	+.448	+.064	+.064	-.384
17	-.048	+.320	+.048	+.048	-.272
18	-.032	+.200	+.032	+.032	-.168
19	-.015	+.081	+.015	+.015	-.076
20	0.000	0.000	0.000	0.000	0.000
21	+.011	-.068	-.011	-.011	+.057
22	+.019	-.115	-.019	-.019	+.096
23	+.024	-.143	-0.024	-.024	+.179
24	+.026	-.154	-.026	-.026	+.128
25	+.025	-.150	-.025	-.025	+.125
26	+.022	-.135	-.022	-.022	+.113
27	+.018	-.108	-.018	-.018	+.091
28	+.013	-.077	-.013	-.013	+.064
29	+.006	-.039	-.006	-.006	+.033
30	0.000	0.000	0.000	0.000	0.000

Section A

Live load
Unif. = (44.95)(0.62)(2) = 57.6
Conc. = (1)(26.0)(2) = 52.0
L.L.(+ Impact) = 109.6 X 1.22 = 133.7 k
Dead load
Unif. = (39.96)(0.787)(7) = D.L. = 220.0 k
Curbs and Railings
Unif. = (39.96)(0.300) = C.R. = 12.0 k
Surface
Unif. = (39.96)(0.150)(7) = S. = 42.0 k
Total = 407.7 k
Per Girder = 58.2 k

Section B

Live load
Unif. = (120)(0.64)(2) = 153.8
Conc. = (1)(26.0)(2) = 52.0
L.L.(+ Impact) = 205.8 X 1.22 = 250.0 k
Dead load
Unif. = (106.7)(0.787)(7) = D.L. = 58.8 k
Curbs and Railings
Unif. = (106.7)(0.300) = C.R. = 32.0 k
Surface
Unif. = (106.7)(0.150)(7) = S. = 112.0 k
Total = 982.0 k
Per Girder = 140.3 k

Section A-8

Live load
Unif. = (42.81)(0.64)(2) = 54.90
Conc. = (0.877)(26.0)(2) = 45.6
L.L.(+ Impact) = 100.5 X 1.22 = 123.0 k
Dead load
Unif. = (40.0)(0.787)(7) = D.L. = 22.0 k
Curbs and Railings
Unif. = (40.0)(0.300) = C.R. = 12.0 k
Surface
Unif. = (40.0)(0.150)(7) = S. = 42.0 k
Total = 397.0 k
Per Girder = 56.7 k

Section A-9

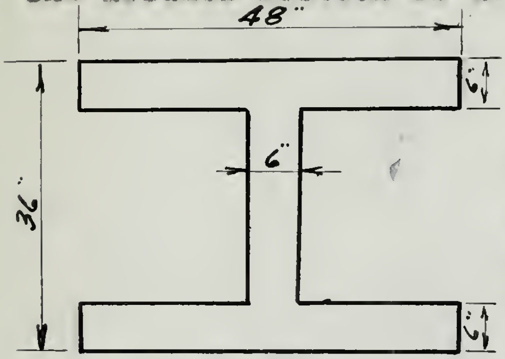
Live load
Unif. = (51.89)(0.64)(2) = 66.6
Conc. = (0.946)(26.0)(2) = 49.1
L.L.(+ Impact) = 115.7 X 1.22 = 141.0 k
Dead load
Unif = (49.9)(0.787)(7) = D.L. = 274.0 k
Curbs and Railings
Unif. = (49.9)(0.300) = C.R. = 15.0 k
Surface
Unif. = (49.9)(0.150)(7) = S. = 52.5 k
Total = 482.5 k
Per Girder= 69.0 k

Section B-11

Live load
Unif. = (48.4)(0.64)(2) = 62.0
Conc. = (0.924)(26.0)(2)= 48.0
L.L.:(+ Impact) = 110.0 X 1.22 = 134.0 k
Dead load
Unif. = (39.72)(0.787)(7) = D.L. = 218.0 k
Curbs and Railings
Unif. = (39.72)(0.300) = C.R. = 11.9 k
Surface
Unif. = (39.72)(0.150)(7) = S. = 41.7 k
Total = 405.6 k
Per Girder = 58.0 k

CROSS SECTION DESIGN

In the design of the cross section there is no exact design for determining the depth of the beam. Therefore when dealing with a section other than rectangular, it is necessary to assume a section. From the concrete properties a section of about 30" is indicated. Since it is known that the eccentricity will control, the first section assumed was one with a depth of 36". The width of the beam has already been assumed as 48" previously. The assumed section is as indicated in Fig. 12.



A = area = 720 sq. in.

I = moment of inertia = 137,968 in^4

$r^2 = I \div A = 191.5$ sq. in.

$r^2/y_t = 10.6$ in.

$w_d = \frac{720 \times 150}{144} = 750$ lb. per ft.

Fig. 12

We determine the limiting values of e' and P_i by constructing a graph for each of the three critical sections - the midpoint of the first span (A), the first interior support (B), and the midpoint of the central span (C). These graphs are the condition equations developed in the section Fundamental Formulae.

Using the equations from the section Fundamental Formulae, we first consider section A.

Section A

The Bending Moments are:

$M_d = +114,800(5) = +574,000$ fp

$M_a = +497,600$ fp

$M'_a = 0$

The stresses in the extreme fibres are:

$$f_{dt} = f_{db} = \frac{574,000 \text{ X } 12 \text{ X } 18}{137,968} = 900 \text{ psi}$$

$$f_{at} = f_{ab} = \frac{497,600 \text{ X } 12 \text{ X } 18}{137,968} = 780 \text{ psi}$$

$$f'_{at} = f'_{ab} = 0$$

In this case,

$$c > f_{dt} + f_{at} \quad \text{as} \quad 2000 > 1680$$

$$c_t > f'_{at} - f_{dt} \quad \text{as} \quad 0 > -900$$

$$c > f'_{ab} - f_{db} \quad \text{as} \quad 2000 > -900$$

Therefore,

$$\text{line(2)}: + \frac{1}{(c - f_{dt} - f_{at})A} = + \frac{1}{(2000 - 1680)A} = + \frac{3.15}{1000A}$$

$$\text{line(2')}: - \frac{1}{(c_t - f'_{at} + f_{dt})A} = - \frac{1}{(0 + 900)A} = - \frac{1.11}{1000A}$$

$$\text{line(4)}: + \frac{n}{(f_{db} + f_{ab} - c_t)A} = + \frac{.85}{(1580 - 0)A} = + \frac{0.5}{1000A}$$

$$\text{line(4')}: + \frac{1}{(c - f'_{ab} - f_{db})A} = + \frac{1}{(2000 + 900)A} = + \frac{0.345}{1000A}$$

With these values the diagram shown in Fig. 13 can be drawn (solid lines).

Section B

The Bending Moments are:

$M_d = -150,300(5) = -751,500$ fp

$M_a = -574,100$ fp

$M'a = 0$

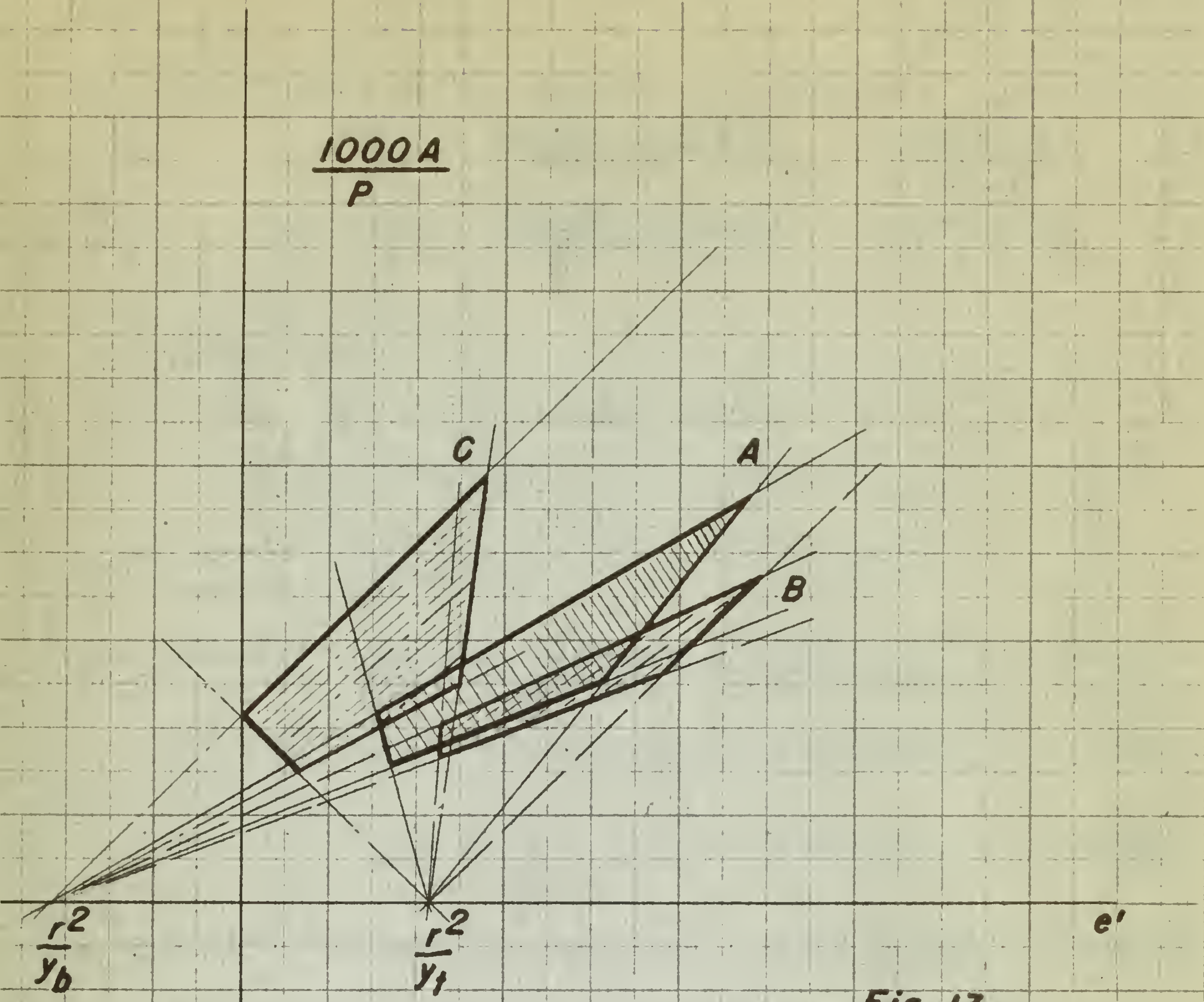


Fig. 13

e' vs. $\frac{1000A}{P}$

36" Depth

The stresses in the extreme fibres are:

$$f_{dt} = f_{db} = \frac{751,500 \times 12 \times 18}{137,968} = 1178 \text{ psi}$$

$$f_{at} = f_{ab} = \frac{574,100 \times 12 \times 18}{137,968} = 900 \text{ psi}$$

$$f'_{at} = f'_{ab} = 0$$

In this case,

$$c < f_{dt} + f_{at} \quad \text{as } 2000 < 2078$$

$$c_t > f'_{at} - f'_{dt} \quad \text{as } 0 > -1178$$

$$c > f'_{ab} - f_{db} \quad \text{as } 2000 > -1178$$

Therefore,

$$\text{line(2)}: \; -\frac{n}{(f_{dt} + f_{at} - c)A} = -\frac{0.85}{(2078 - 2000)A} = -\frac{11.3}{1000A}$$

$$\text{line(2')}: \; -\frac{1}{(c_t - f'_{at} + f_{dt})A} = -\frac{1}{(0 + 1178)A} = -\frac{0.85}{1000A}$$

$$\text{line(4)}: \; +\frac{n}{(f_{db} + f_{ab} - c_t)A} = +\frac{0.85}{(2078 - 0)A} = +\frac{0.41}{1000A}$$

$$\text{line(4')}: \; +\frac{1}{(c - f'_{ab} + f_{db})A} = +\frac{1}{(2000 + 1178)A} = +\frac{0.32}{1000A}$$

With these values the diagram shown in Fig 13 can be drawn (dashed lines).

Section C

The Bending Moments are:

$$M_d = +39,000(A) = +196,000 \text{ fp}$$

$$M_a = +332,000 \text{ fp}$$

$$M'_a = -87,300 \text{ fp}$$

In this case, the stresses in the extreme fibres are:

$$f_{dt} = f_{db} = \frac{196,000 \times 12 \times 18}{137,968} = 307 \text{ psi}$$

$$f_{at} = f_{ab} = \frac{332,000 \times 12 \times 18}{137,968} = 521 \text{ psi}$$

$$f'_{at} = f'_{ab} = \frac{87,300 \times 12 \times 18}{137,968} = 137 \text{ psi}$$

In this case,

$$c > f_{dt} + f_{at} \quad \text{as } 2000 > 828$$

$$c_t > f'_{at} - f_{dt} \quad \text{as } 0 > -170$$

$$c > f'_{ab} - f_{db} \quad \text{as } 2000 > -170$$

Therefore,

$$\text{line(2):} \quad + \frac{1}{(c - f_{dt} - f_{at})A} = + \frac{1}{(2000 - 828)A} = + \frac{0.854}{1000A}$$

$$\text{line(2'):} \quad - \frac{1}{(c_t - f'_{at} + f_{dt})A} = - \frac{1}{(0 + 170)A} = - \frac{5.9}{1000A}$$

$$\text{line(4):} \quad + \frac{0.85}{(f_{db} + f_{ab} - c_t)A} = + \frac{0.85}{(828 - 0)\ A} = + \frac{1.03}{1000A}$$

$$\text{line(4'):} \quad + \frac{1}{(c - f'_{ab} + f_{db})A} = + \frac{1}{(2000 + 170)A} = + \frac{0.46}{1000A}$$

With these values the diagram shown in Fig. 13 can be drawn (dotted and dashed lines).

With a 36" section the maximum e obtainable allowing 3" for jacking purposes is 9". We assume e_A = 9" which is less than the r^2/y_t = 10.6" putting the wire within the core of the section. For e_A = 9", the maximum value of e'_B - the controlling eccentricity - is 8.5". Examining the diagram of $1/P_i$ vs e'_B, we find that the value cannot be satisfied. Hence the depth of beam section must be increased, The next section to try has a depth of 42".

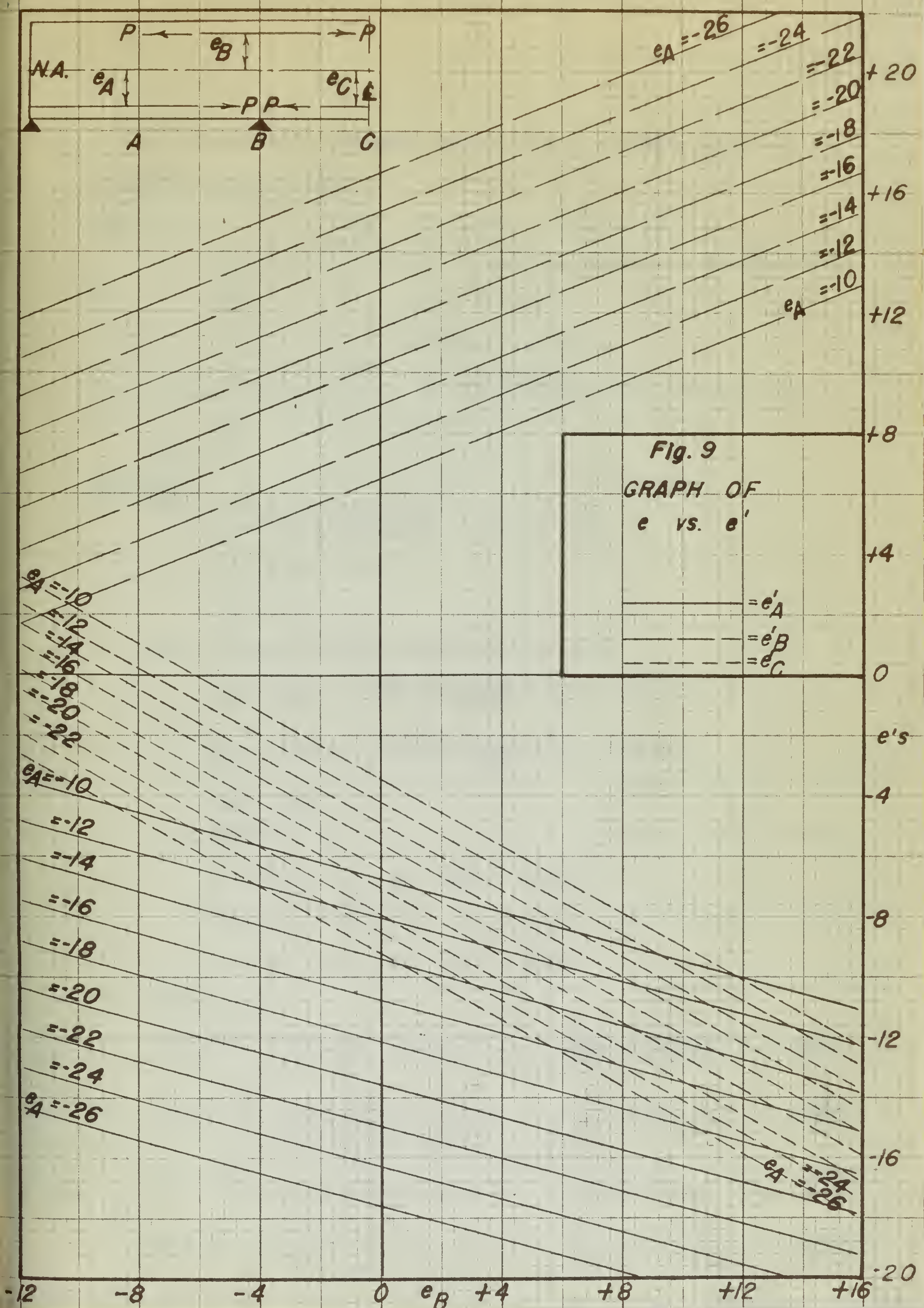
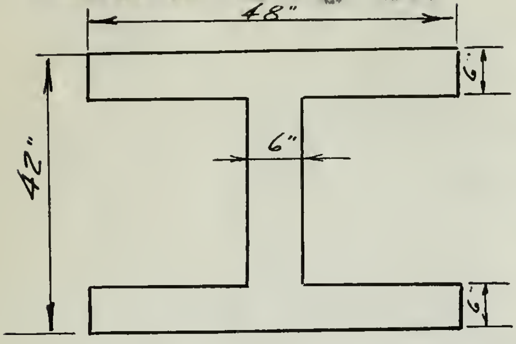


Fig. 9

GRAPH OF

e vs. e'

A section with a depth of 42" is now assumed - as indicated in Fig. 14.



A = area = 756 sq. in.

I = moment of inertia = 201,852 in^4

$r^2 = 267$ in^2

$r^2/y_t = 12.72$ in.

$w_d = \dfrac{756 \times 150}{144} = 787$ lb. per ft.

Fig. 14

Section A

The Bending Moments are:

$M_d = +114{,}800(5.25) = +603{,}000$ fp

$M_a = +497{,}600$ fp

$M'_a = 0$

The stresses in the extreme fibres are:

$$f_{dt} = f_{db} = \frac{603{,}000 \times 12 \times 21}{201{,}852} = 753 \text{ psi}$$

$$f_{at} = f_{ab} = \frac{497{,}600 \times 12 \times 21}{201{,}852} = 622 \text{ psi}$$

$$f'_{at} = f'_{ab} = 0$$

In this case,

$c > f_{dt} + f_{at}$ as $2000 > 1375$

$c_t > f'_{at} - f_{dt}$ as $0 > -753$

$c > f'_{ab} - f_{db}$ as $2000 > -753$

Therefore,

$$\text{line(2): } +\frac{1}{(c - f_{dt} - f_{at})A} = + \frac{1}{(2000 - 1375)A} = + \frac{1.6}{1000A}$$

$$\text{line(2'): } -\frac{1}{(c_t - f'_{at} + f_{dt})A} = - \frac{1}{(0 + 753)A} = - \frac{1.33}{1000A}$$

$$\text{line(4): } + \frac{n}{(f_{db} + f_{ab} - c_t)A} = + \frac{0.85}{(1375 - 0)A} = + \frac{0.618}{1000A}$$

$$\text{line(4'): } +\frac{1}{(c - f'_{ab} + f_{db})A} = + \frac{1}{(2000 + 753)A} = + \frac{0.362}{1000A}$$

With these values the diagram shown in Fig. 15 can be drawn (solid lines).

Section B

The Bending Moments are:

$$M_d = -150{,}300(5{,}25) = -789{,}000 \text{ fp}$$

$$M_a = -574{,}100 \text{ fp}$$

$$M'_a = 0$$

The stresses in the extreme fibres are:

$$f_{dt} = f_{db} = \frac{789{,}000 \times 12 \times 21}{201{,}852} = 985 \text{ psi}$$

$$f_{at} = f_{ab} = \frac{574{,}100 \times 12 \times 21}{201{,}852} = 718 \text{ psi}$$

$$f'_{at} = f'_{ab} = 0$$

In this case,

$$c > f_{dt} + f_{at} \quad \text{as} \quad 2000 > 1703$$

$$c_t > f'_{at} - f_{dt} \quad \text{as} \quad 0 > -985$$

$$c > f'_{ab} - f_{db} \quad \text{as} \quad 2000 > -985$$

Therefore,

$$\text{line(2):} \quad + \frac{1}{(c - f_{dt} - f_{at})\,A} = + \frac{1}{(2000-1703)A} = + \frac{3.37}{1000A}$$

$$\text{line(2'):} \quad - \frac{1}{(c_t - f'_{at} + f_{dt})A} = - \frac{1}{(0 + 985)_A} = - \frac{1.02}{1000A}$$

$$\text{line(4):} \quad + \frac{n}{(f_{db} + f_{ab} - c_t)A} = + \frac{0.85}{(1703 - 0)A} = + \frac{0.50}{1000A}$$

$$\text{line(4'):} \quad + \frac{1}{(c - f'_{ab} + f_{db})A} = + \frac{1}{(2000 + 985)} = + \frac{0.335}{1000A}$$

With these values the diagram shown in Fig. 15 can be drawn (dashed lines).

Section C

The Bending Moments are:

$$M_d = +39{,}300(5.25) = +206{,}500 \text{ fp}$$

$$M_a = \cancel{+}\ 322{,}100 \text{ fp}$$

$$M'_a = -87{,}300 \text{ fp}$$

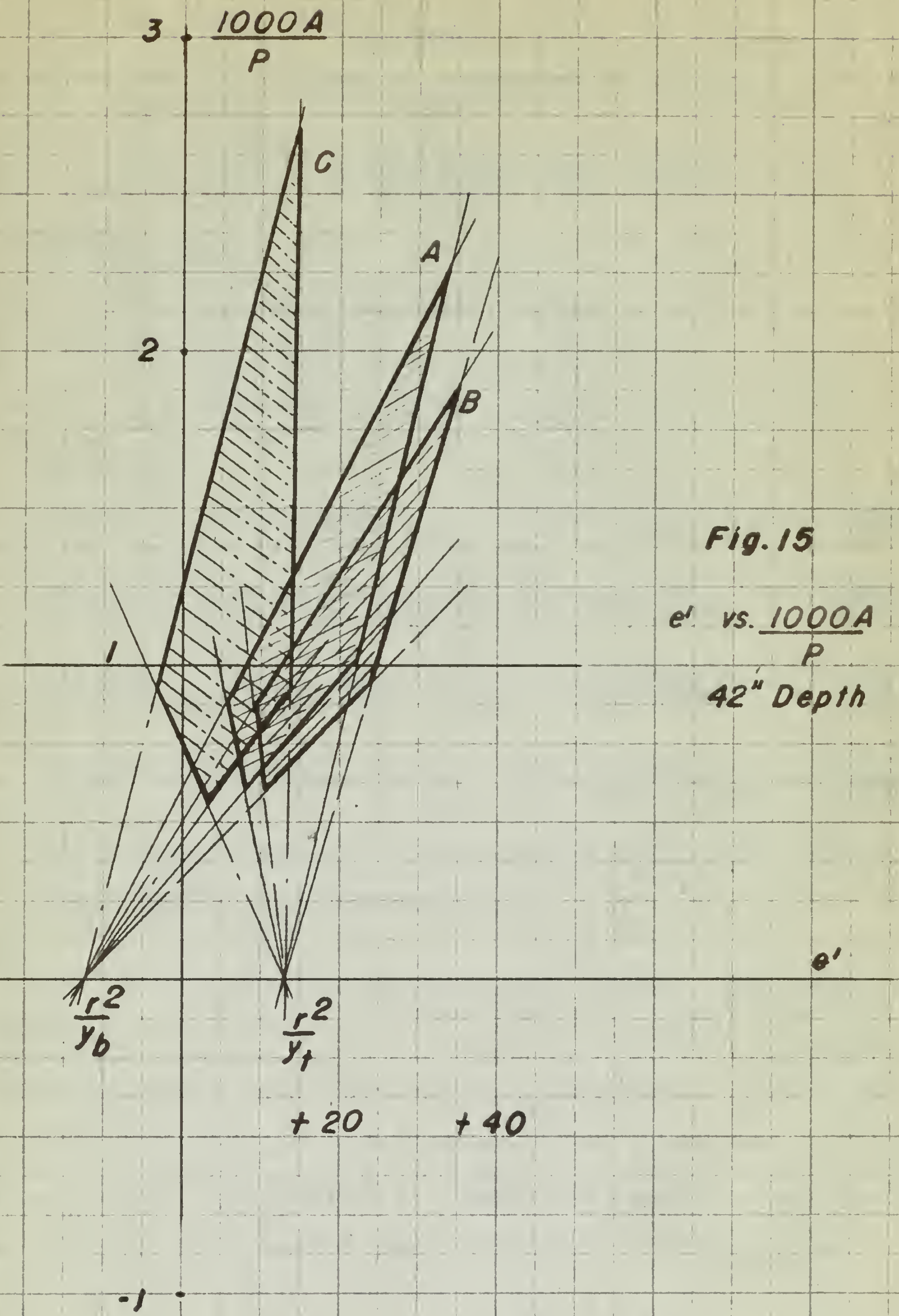


Fig. 15

e' vs. $\frac{1000A}{P}$

42" Depth

The stresses in the extreme fibres are:

$$f_{dt} = f_{db} = \frac{206{,}500 \text{ X } 12 \text{ X } 21}{201{,}852} = 258 \text{ psi}$$

$$f_{at} = f_{ab} = \frac{322{,}100 \text{ X } 12 \text{ X } 21}{201{,}852} = 401 \text{ psi}$$

$$f'_{at} = f'_{ab} = \frac{87{,}300 \text{ X } 12 \text{ X } 21}{201{,}852} = 109 \text{ psi}$$

In this case,

$$c > f_{dt} + f_{at} \quad \text{as } 2000 > 659$$

$$c_t > f'_{at} - f_{dt} \quad \text{as } 0 > -149$$

$$c > f'_{ab} - f_{db} \quad \text{as } 2000 > -149$$

Therefore,

$$\text{line(2): } + \frac{1}{(c - f_{dt} - f_{at})A} = + \frac{1}{(2000-659)} = + \frac{0.746}{1000A}$$

$$\text{line(2'): } - \frac{1}{(c_t - f'_{at} + f_{dt})A} = - \frac{1}{(0 + 149)A} = - \frac{6.72}{1000A}$$

$$\text{line(4): } + \frac{n}{(f_{db} + f_{ab} - c_t)A} = + \frac{0.85}{(659 - 0)A} = + \frac{1.29}{1000A}$$

$$\text{line(4'): } + \frac{1}{(c - f'_{ab} + f_{db})\,A} = + \frac{1}{(2000 + 149)A} = + \frac{0.443}{1000A}$$

With these values the diagram shown in Fig. 15. can be drawn (dotted and dashed lines).

Fig. 15 represented the graphical solution satisfying all the conditions for the compressive and tensile stresses in the concrete at the three critical sections - at time of prestress and after elapsed loading (i.e., after the prestress force has been decreased to n times its original value due to the creep of the concrete and steel and the plastic flow of the concrete).

With graph of e' versus 1/P which defines according to the condition equations the limits of e at the critical sections, and the graph of e' versus e to compensate for the secondary bending moment, it is possible to determine the values of the actual eccentricity. Since straight cables are used the cables must be within the core of the section at the exterior supports to prevent tension in the top fibre. Therefore, e_A must be less than r^2/y_t which equals 12.7". e_A was assumed equal to -12". This allows 3" clearance beneath the flanges for jacking purposes. Since it is desirable to $\overset{\text{HAVE}}{\wedge}e_B$ as large as possible, the first try was e_B equal to +12". With $e_A = -12$, and $e_B = +12$, the values of the equivalent eccentricities (e') from the graph of e versus e' are:

$e'_A = -11.6$, $e'_B = +12.6$, and $e'_C = -11.2$

Entering the graph of e' versus 1000A/P - the highest value of 1000A/P which will satisfy the above e's is equal to 1. The limiting values of the equivalent eccentricities with 1000A/P equal to 1 are:

$e'_A = -22$ to -8

$e'_B = +24.5$ to 12.5

$e'_C = -13.5$ to 0.

Since the actual values of e satisfy both graphs, the assumed values of e_A and e_B are acceptable. Therefore, the values of eccentricity used are:

$e_A = -12"$, $e_B = +12"$ and $e_C = e_A = -12"$.

With $\frac{1000 \text{ x } A}{P} = 1$

$$P = 1000 \text{ x } A = 1000 \text{ x } 756 = 756{,}000 \text{ lb.}$$

With the value of $P = 756{,}000$ and the value of $e = \pm 12$ we have the following stresses:

$$\frac{P}{A} = \frac{756{,}000}{756} = +\ 1000 \text{ psi}$$

$$f_p = \frac{Pe \text{ x } c}{I} \quad \frac{756{,}000 \text{ x } 12 \text{ x } 21}{201{,}852} = \pm 944 \text{ psi}$$

$$\text{Sec. Mom.} = -P\ [0.6375\ (e_a) + 0.5625\ (e_b)]$$

$$= -756{,}000\ [0.6375\ (-12) + 0.5625\ (+12)]$$

$$= -756{,}000 \text{ x } 12\ (-.6275 + 0.5625)$$

$$= -756{,}000 \text{ x } 12\ (-0.075) = +\ 680{,}000 \text{ in. lb.}$$

$$fs = \frac{680{,}000 \text{ x } 21}{201{,}852} = \pm 71 \text{ psi}$$

Since the condition equations have been satisfied and the diagrams for secondary moments have also been complied with, it is reasonable to assume that the stresses at the critical sections are not above the allowable stress. However, for a check we have tabulated the stresses at these sections.

	A		B		C	
	Top	Bottom	Top	Bottom	Top	Bottom
Dead Load Stress	+662	-662	-718	+718	+401	-401
Live Load (super)	+753	-753	-985	+985	+258	-258
P/A stress	+1000	+1000	+1000	+1000	+1000	+1000
Pec/I	-944	+944	+944	-944	-944	+944
f_s	+ 36	-36	+71	-71	+71	-71
Resulting Stress	+1507	+493	+312	+1688	+786	+1214
Stress with no live load	+845	+1155	+1030	+970	+528	+1472
Total Stress After Elapsed Time	+1500	+202	+21	+1680	+778	+923

DETERMINATION OF WIRE AREA

The following computations are to determine the wires required.

Design stress = 120,000 lb. per sq. in.
Area = 0.05983
D = 0.276"
P = 756,000 lb.

Area of steel = $\frac{756,000}{120,000}$ = 6.29 sq. in.

Number of wires = $\frac{6.29}{0.05983}$ = 105

Therefore, use 4 cables of 28 wires each (total of 112) placed two cables on each side of the web.

Load per wire = $\frac{756,000}{112}$ = 6750 lb., less than allowable 7200 lb.

Size of Sheath

Spacing = 1 diameter (to allow for grouting)

Wires - 7 by 4

Sheath - (14)(0.276) by (8)(0.276) or 5-7/8" by 2-1/4"

Use 4 standard sandwhich plates for each cable. Total size for each cable is 4" by 4-1/2".

DESIGN OF BEARING PLATES

LONGITUDINAL WIRES

Allowable Bearing Value = 1.75 f_c = 1.75 x 2000 = 3500 psi

Loss of concrete for cables = 2(3.875 x 2.25) = 17 sq. in.

Required Area (one plate each side of web)

$$A = \frac{756,000}{2 \times 3500} = 108 \text{ sq. in.}$$

Use plate 6" x 21" = 126" (greater than 108 + 17)

Plate Thickness

In the design of the plate thickness, the design as suggested in AISC for column bearing plate.

$$t^2 = 0.15\ pm^2$$

$$= 0.15(3.5)\ (3)^2 = 4.72$$

$$t = 2.18"$$

use t = 2-1/2"

SHEAR CHECK

The shear stress or diagonal tensile stress (v) in an uncracked concrete section is maximum at the centroid. For a girder subject to bending only, the value of v may be computed from the following equation which is a standard textbook formula for maximum shearing stress in a homogeneous section.

$$v = \frac{VQ}{bI}$$

in which, v = shearing stress at centroid

V = external shear on section

b = width of section at centroid

Q = statical moment of section on either side of centroid taken about that point

I = moment of inertia of entire section about the centroid

When the prestressing wires run through straight from end to end, as they do in this girder design, the prestress forces are parallel to the girder axis and do not contribute to the value of V. However, the prestress force exerts a horizontal compressive stress (S_x). The stress v and S_x produce a principle tensile stress which may be computed from the following standard textbook formula.

$$S_t = \frac{1}{2}\sqrt{(\quad 4v^2 + S_x^2} - S_x)$$

If the principle stresses exceed the allowable tensile strength of the concrete, web reinforcement is necessary.

PRINCIPLE STRESSES

Section A

$V = 58.2$ k
$b = 6"$
$Q = (48)(6)(18) + (6)(15)(7.5) = 5855 \text{ in}^3$
$I = 201{,}852 \text{ in}^4$

$$v = \frac{(58.2)(5855)}{(6)(201{,}852)} = 282 \text{ psi}$$

$$S_x = nP_i/A = (0.85)(756{,}000)/(756) = 850 \text{ psi}$$

$$S_t = \frac{1}{2}\left(\sqrt{4(282)^2 + (850)^2} - 850\right) = 85 \text{ psi}$$

Section B

$V = 140.3$
$b = 6"$
$Q = 5855$
$I = 201{,}852$

$$v = \frac{(140.3)(5855)}{(6)(201{,}852)} = 682 \text{ psi}$$

$S_x = 850$ psi (constant throughout the length of beam)

$$S_t = \frac{1}{2}\left(\sqrt{4(682)^2 + (850)^2} - 850\right) = 382 \text{ psi}$$

since 382 psi exceeds the allowable strength of 120 psi, web reinforcement in the form of vertical stirrups is necessary.

Section A-8

$$v = \frac{(56.7)(5855)}{(6)(201{,}852)} = 274 \text{ psi}$$

$S_t = 84$ psi

Section A-9

$$v = \frac{(69.0)(5855)}{(6)(201{,}852)} = 334 \text{ psi}$$

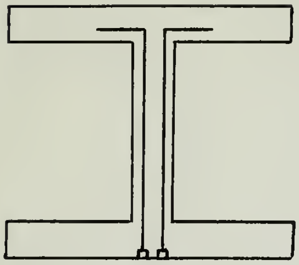
$S_t = 119$ psi

Section B-11

$$v = \frac{(58.0)(5855)}{(6)(201{,}852)} = 281 \text{ psi}$$

$S_t = 81$ psi

WEB REINFORCEMENT

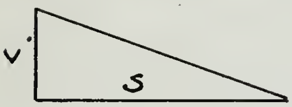


Shear strength of concrete $= 0.02\ f'_c$
$= 0.02\ (6000$
$= 120$ psi

Reinforcement must provide all in excess of 120 psi.

Since this shearing stress drops rapidly in either direction of the interior support, reinforcement is theoretically not required at a distance greater than 10' from the interior support. To provide the reinforcement, stirrups will be used. The design of these stirrups follows the design of stirrups as outlined in the Reinforced Concrete Design Handbook.

Stirrup Design



Max $V' = 262$ psi
$S = 10'$
$b = 6"$
$d = 42"$
$f_v = 20{,}000$ psi
$f'_c = 6000$ psi

Try 3/8 inch stirrups:

$A_v f_v = 4400$

$$\text{Max } (1/s) = \frac{(\text{Max } v')b}{A_v f_v} = \frac{262 \times 6}{4400} = 0.375$$

$N = 6S(\max 1/s) = 6 \times 10 \times 0.375 = 22.5$, say 24

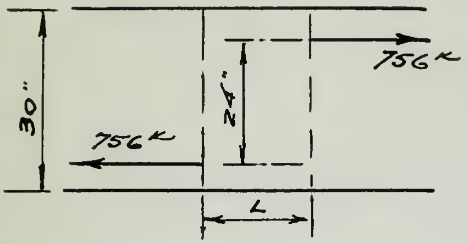
$\text{Index} = 1.5S/(\max 1/s) = 1.5(10)/0.375 = 40$

Entering "Spacing of Stirrups" diagram, page 81, RCDH, with index of 40 and max(1/s) of 0.375, we get:

3/8" round stirrups spaced: 10 at 3"; 7 at 4"; 3 at 6"; 1 at 8"; and 3 at 12".

This spacing will commence at the interior supports and run in both directions. Although the stirrups are theoretically not required after the 10', it was decided to continue the stirrups throughout the length of the beam at the maximum spacing of 12".

DESIGN OF ANCHOR BLOCKS



Since the designed cable is discontinuous, it is necessary to have anchoring blocks at the points of discontinuity. Therefore, at the 3/4 point in the first span a rectangular block is placed on both sides of the web of the I beam. The cable which is below the neutral axis ends at this block while the cable over the first interior support begins. There is a similar block at the 1/4 point of the middle. Throughout the entire length of the span there are four such blocks. The prestressing cables bearing on these blocks produce an internal couple of 756,000 x 24 in. lb. This couple is resisted by shear along the flange. Although the web aids in resisting this couple, it has not been taken into account and the design is on the side of safety.

Length of Block

$$\text{Shear} = \frac{24 \times 75^6}{30} = 604.8 \text{ K}$$

$$\text{Shear/in.} = \frac{604.8}{42} = 14.4 \text{ k/in.}$$

If no reinforcement were used, the allowable shear from the ACI standards would be:

$$V_c = 0.02 f'_c = .02 \times 6000 = 120 \text{ psi}$$

$$L = \frac{14.4}{.120} = 120" = 10' \text{ - too large}$$

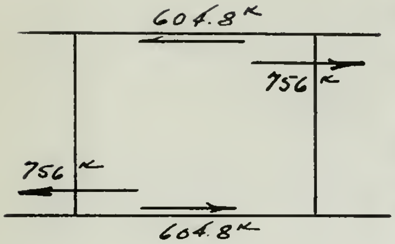
With reinforcement the allowable shear from the ACI standards:

$$V_c = 0.06 f'_c = .06 \times 6000 = 360 \text{ psi}$$

$$L = \frac{14.4}{.360} = 40"$$

Therefore, a block with a length of 4'0" was used.

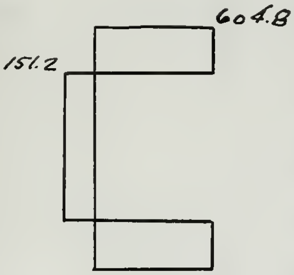
ANCHOR BLOCK REINFORCEMENT



In the design of the block it is noted that the shear is horizontal and hence the main reinforcement is horizontal. Since the actual stress pattern is to a certain extent indeterminate, it was decided to run the reinforcement in both direction. To provide for reinforcement in both direction, a wire mesh cage prefabrication to be lowered into the forms for the anchor blocks will be designed.

Design of Reinforcement

The shear diagram for the block is shown and the computations follow.



Shear taken by the concrete equals the shear value of the concrete times the area effective in shear (length of the block times the width of the block).

$v_c = 0.02(6000)(48" \times 42") = 242$ k

The remainder of the shear must be taken by the reinforcement and is equal to $v_r = 604.8 - 242 = 362.8$ k

We now will assume 6 vertical rows of horizontal reinforcement placed 3 on either side of the centerline (the cable is therefore placed between two rows of reinforcement). Each vertical row will take 362.8 k ÷ 6 rows, or 60.5 k per row.

Now, $A_v = \frac{V' s}{f_v j d}$

$$= \frac{60.5 s}{20(7/8)(30)}$$

$$= 0.115 s$$

where V' = excess of total shear over that permitted on the concrete (v_r)

A_v = total area of reinforcement in tension within a distance s

s = spacing of reinforcement in a direction parallel to it

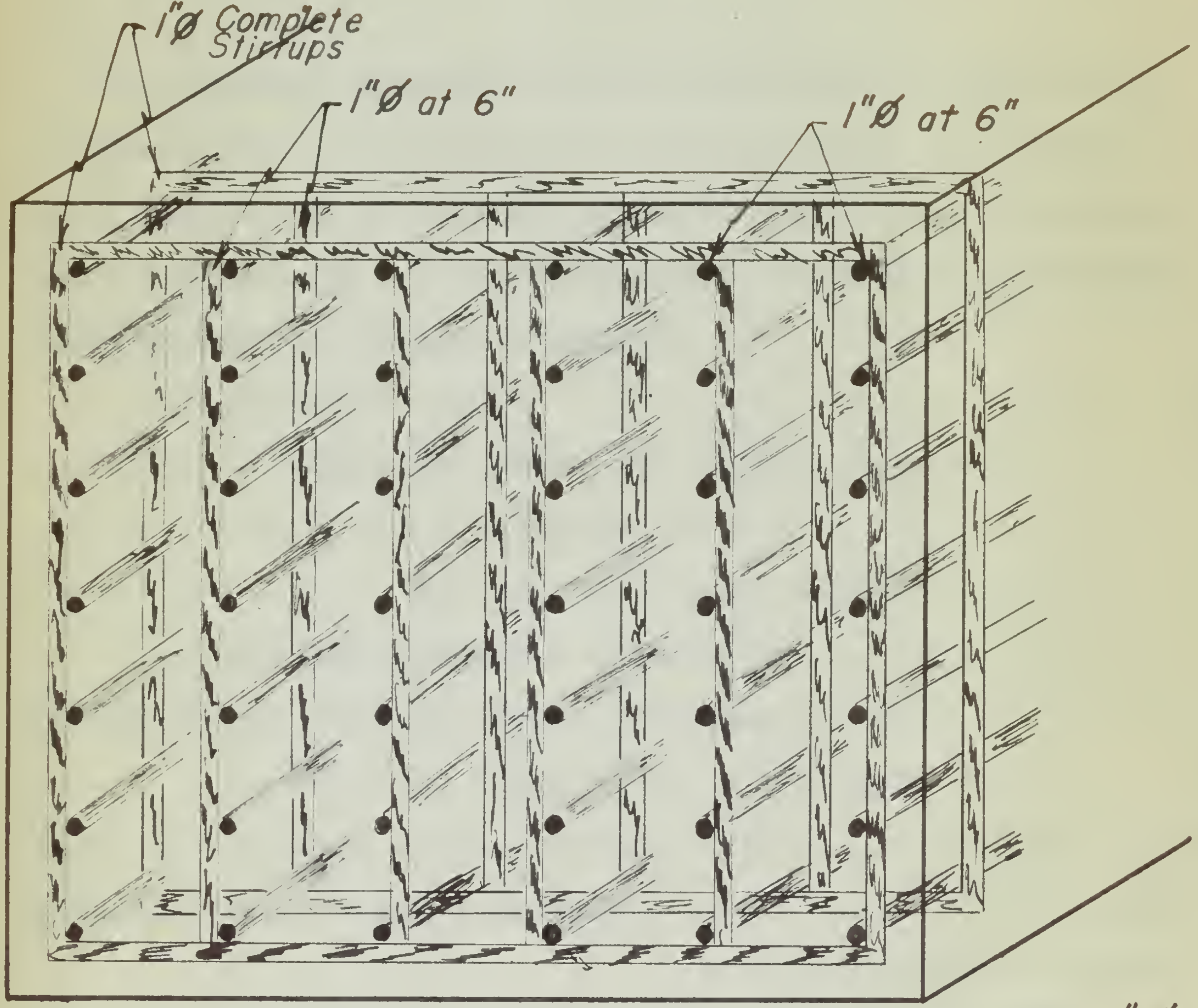
f_v = tnesile unit stress in reinforcement

Assume a 6" spacing of reinforcement with a required A_v of 0.690 in^2. Use 1" Ø spaced 6" in both directions.

The wire mesh cage would therefore be fabricated as follows:

- -6 vertical rows of 1" Ø spaced 6" vertically
- -vertical 1" Ø spaced 6" horizontally
- -the vertical 1" Ø would be the vertical leg of a rectangular closed stirrup serving to tie the unit together
- -all bars would be welded so as to form the wire cage

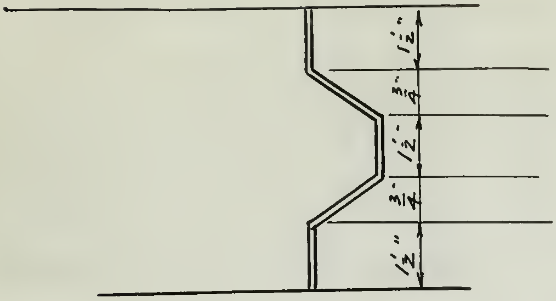
This is illustrated in Fig. 17.



All bars 1" Ø & welded to form a cage.

Fig. 17
ANCHOR BLOCK REINFORCEMENT

DESIGN OF SHEAR KEY



To have an efficient transfer of stress between adjacent girders it is necessary to key them at the upper and lower flanges.

The greatest concentration of load midway between girder centers at the shear key will be equal to that exerted by a wheel loading of 16,000 lbs. placed either side of the shear plane. This load can be distributed according to Art 3.3.2, AASHO Standards, 1949, as follows:

$$E = 0.6S + 2.5$$

where E = width over which wheel load distributed

S = effective span length

$$E = 0.6(4) + 2.5 = 4.9 \text{ feet}$$

Therefore, the maximum shear stress equals

$$16{,}000 \div 4.9 = 3{,}270 \text{ p/ft} = 3.27 \text{ k/ft}$$

The shear strength of the concrete = $0.02\ f_c$ = 120 psi = 0.12 ksi

providing total shear resistance = 2 X 0.120 ksi X 12 in/ft

= 2.88 k/ft/inch of depth

Therefore, the minimum depth of key = 3.27/2.88 = 1.13"

For this design use depth of key = 1-1/2".

TRANSVERSE WIRE

In order to insure that the girders act together as a monolithic structure it is necessary to induce a compressive stress in the upper and lower flanges. This stress is produced by high tension transverse wires which run through the upper and lower flanges and are prestressed in a manner similar to that used in the longitudinal prestressing operation. However, there is no design for the wires for they take no actual load. It was believed that 8 wires of diameter 0.276" in the top flange and another 8 in the bottom flange spaced at 5 feet intervals would be sufficient. With this arrangement the stress produced would be:

Total wire area = 8 X 0.05983 = 0.487 sq. in.

Force = 0.487 X 120,000 = 57,500 lb.

Stress = 57,500/(6 X 60) = 160 psi

The designed wires would result in a compressive stress of 160 psi in both the upper and lower flanges. It is believed that this would be adequate to produce a monolithic structure.

DESIGN OF BEARING PLATES

TRANSVERSE WIRES

Allowable Bearing Value = $1.75\ f_c = 1.75 \times 2000 = 3500$ psi

Loss of concrete for cables = $2\text{-}1/4 \times 1 = 2.25$

Required area = $\frac{8 \times 0.05982 \times 120{,}000}{3{,}500} = 16.4$ sq. in.

Use plate 6" x 4" (greater than 16.4" + 2.25")

Plate Thickness

$$t^2 = 0.15m^2$$

$$= 0.15(3.5)\ (1.5)^2 = 1.18$$

$$t = 1.09"$$

use t = 1-1/4"

MOMENT AT FIRST CRACK

It is generally advisable to determine the moment at which the first crack (M_{cr}) may be anticipated. This involves estimating the stress f_{cr} at which the allowable tensile stress of the concrete is reached in the bottom fibre. The moment at this point may be determined from the following equation:

$$M_{cr} = M_t + f_{cr} \frac{I}{y_b}$$

M_t = total maximum moment = 1,363,000 fp

f_{cr} = allowable tensile stress = 700 psi

I = moment of inertia

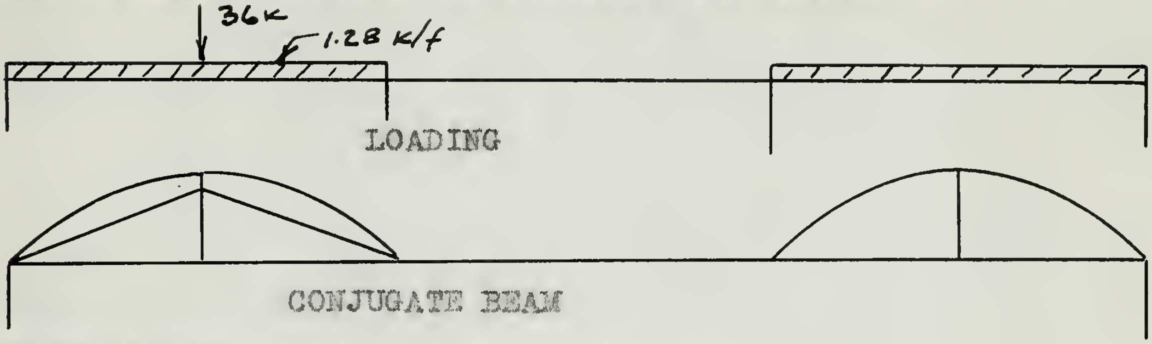
y_b = distance to bottom fibre = 21"

$$M_{cr} = 1{,}363{,}000 \times 12 + 700 \times \frac{201{,}852}{21} = 23{,}300{,}000 \text{ ip}$$

On the assumption that moments and loads are proportional, the calculation shows that the first crack appears when the superimposed load is 42% more than the combined dead load and live load. Unlike ordinary reinforced concrete, this crack will close up when the load is subsequently reduced down to design live load. The concrete will then again behave as a homogeneous material.

DEFLECTION

The ratio of depth to span is about 1:30 which is small for a bridge carrying highway loading as heavy as H20-S16. The deflection under live load including impact should therefore be investigated. The maximum deflection for the beam will occur in the first span when loaded as shown in the sketch. The point of maximum deflection is not at the midspan but for simplicity this deflection will be determined. The method of Conjugate Beam is used to obtain the necessary deflection.



COMPUTATIONS:

$$Pab/L = (36)(1.22)(50)(50)/100 = 1098$$

$$wL^2/8 = (1.28)(1.22)(100^2)/8 = 1953$$

$$M_B = (669.3-165.9)(1.28)(1.22) + (36)(1.22)(10.0) = 1224$$

For Conjugate Beam,

$$R_L = \frac{1}{100} \left(\frac{(1098)(100)(50)}{2} + \frac{(4)(1953)(50)(50)}{3} - \frac{(1224)(100)(100}{(2)(3)} \right.$$

$$= 27,400 + 65,100 - 20,400 = 72,100$$

$$M = (72,100)(50) + \frac{(612)(50)(50)}{(2)(3)} - \frac{(1098)(50)(50)}{(2)(3)} - \frac{2(1953)}{(3)(8)}50^{2}_{3}$$

$$= 3,605,000 + 255,000 - 458,000 - 1,220,000 = 2,070,000$$

$$= M/EI = \frac{2,070,000 \text{ X } 1728}{3,000 \text{ X } 7 \text{ X } 201,853} = 0.845"$$

The live load deflection is approximately 1/1400 of the span. The AASHO requires that the deflection shall not exceed 1/800 of the span. Since the maximum deflection will only be slightly greater than the live load deflection computed, the AASHO requirements are easily satisfied.

COMPARISON OF SECTION DESIGNED

STRAIGHT CABLES
VERSUS
PARABOLIC CABLE

In the design of a three span continuous girder Magnel uses a continuous parabolic cable. The cable sags in the middle of the spans and humps over at the interior supports. The eccentricities of this cable are limited as great differences in the values of e at the midspans and supports would mean a cable of such tortuous shape that the friction between the cable and sheath might be serious. In most instances the cable at the supports doe not cross the neutral axis. Such a cable results in very high secondary bending moments. Straight cables which can be placed equidistant from the neutral axis tend to cancel out on the secondary bending moment.

In the solution of his parabolic cable, Magnel has evolved a complicated graph from which he can determine the minimum and maximum values of the actual eccentricities. It was believed that the results of a girder design using Magnel's cable would make an interesting comparison with the girder already designed.

Since a comparison is to be made a section with the same dimensions as the final section design using the straight cables was selected. This section has a 42" depth and the curves of e' versus $1/P_i$ are as before in Fig. 15. Entering this curve with $1000A/P = 1$,

the limiting values as before are:

$e'_A = -22$ to -8

$e'_B = +24.5$ to $+12.5$

$e'_C = -13.5$ to 0

Using Magnel's diagram as shown in Fig. 16 where for an assigned value of e_B the corresponding values of e_A and e_C are obtained provided that a cross-hatched area which satisfies the limits for the equivalent eccentricities is obtained. We have assumed that e_B must be less than +4 in order to prevent excessive friction. With that limitation it appears that such a cross-hatched area is not obtained and therefore the line 1000A/P = 1 is unsuitable. Lower values of 1000A/P did not produce a solution. Therefore the depth would have to be increased. If the eccentricity was not limited by the friction consideration, Magnel's design would result in the same depth of section. The straight cables therefore have advantages in some cases over the parabolic cable. It is true that the eccentricity of the straight cables is limited by the necessary clearance for "jacking" purposes. However, enough eccentricity is obtainable to give an economical section which is smaller than the corresponding section using a parabolic cable.

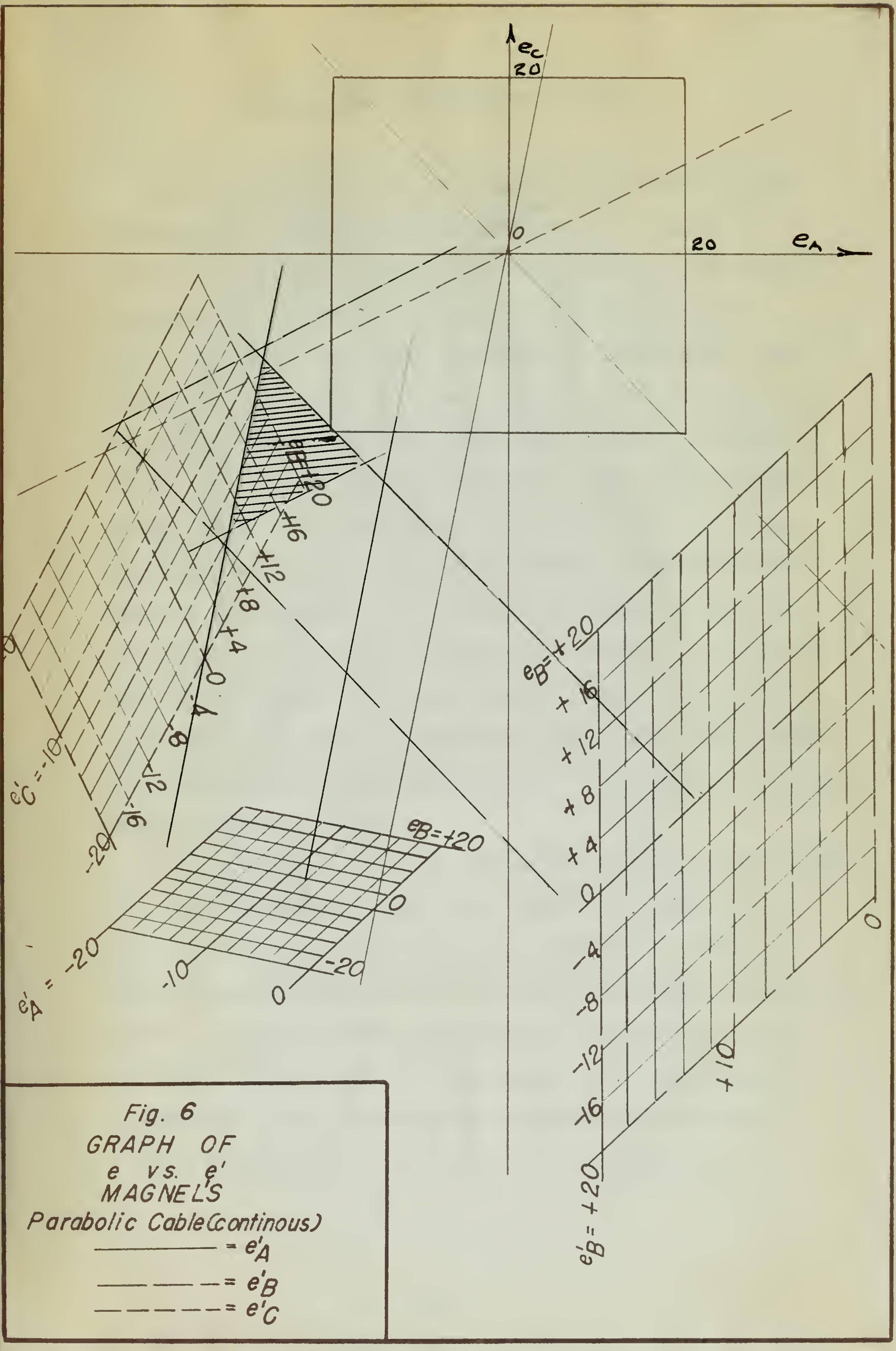


Fig. 6

GRAPH OF

e vs. e'

MAGNEL'S

Parabolic Cable(continous)

——————— = e'_A

——— ——— — = e'_B

— — — — — — = e'_C

CONCLUSIONS AND RECOMMENDATIONS

In the design of the three equal span girder bridge discontinuous straight cables arranged in a manner similar to ordinary reinforcement were used. This design proved to be entirely satisfactory resulting in zero tensile stresses under any condition of loading and gave a depth of span ratio of approximately 1/30. In comparing the design with a similar design using Magnel's parabolic cable it was shown that the parabolic cable necessitated a greater depth of section because of the friction between the sheath and cables which limited the eccentricities. Hence it is concluded that the use of straight cables in the design of continuous bridges is practical and in many cases would result in an economical design. In addition the design procedure is relatively simple and substitutes a simple graph for the complex graph needed in the solution using a parabloic cable.

It is recommended that the use of discontinuous cables for other than three equal span continuous beams be investigated and design procedure modified to apply to continuous bridges of 2 or 3 spans - equal and unequal spans. It is further recommended that the use of the new high strength prestressing bars recently developed in England be investigated in connection with these designs employing discontinuous straight cables.

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